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Thermal entanglement in three-qubitIzing model with nearest and next-nearestneighbor interactionswith added Dzyaloshinskii-Morriya interaction at low temperatures

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ABSTRACT: Most quantum entanglement investigations are focused on two qubits or some finite (small) chain structure, since the infinite chain structure is a considerably cumbersome task. Therefore, the quantum entanglement properties involving an infinite chain structure is quite important,not only because the mathematical calculation is cumbersome but also because real materialsare well represented by an infinite chain. In this paper we consider a system of 3-spin ½ particles inIzing model with nearest and next-nearest neighbor interactions with added Dzyaloshinskii-Morriya interaction at low temperatures. Lack of inversion symmetry in the presence of the DM interaction stimulated us to study entanglement of the three-qubit through symmetric(SY) and non-symmetric (NYS) ways and calculate the thermal entanglement (concurrence) of any two spins as a function of DM interaction and temperature andfind some intriguing result.
Key words:Izing model,qubit-Dzyaloshinskii-Morriya interaction-thermal entanglement-concurrence

INTRODUCTION

Quantum entanglement is one of the most attractive types of correlations that can be shared only among quantum systems (Amico , Fazio , Osterloh , Vedral. 2008). In recent years, many efforts have been devoted to characterize qualitatively and quantitatively the entanglement properties of condensed matter systems, which are the natural candidate to apply for quantum communication and quantum information. In this sense, it is very important to study the entanglement of solid state systems such as spin chains(Anderson, 1958,Yu , Song. 2005). The Heisenberg chain is one of the simplest quantum systems, which exhibits entanglement; due to the Heisenberg interaction it is not localized in the spin system.The entanglement ofthermal states was introduced. Its properties, including threshold temperature, magnetic field dependence andanisotropic effects, were studied. The pairwise entanglement in one-dimensional infinite-lattice anisotropic model was introduced.(Coffman,Kundu,Wootters,2000.) The bipartite entanglementis well understood, while the multipartite entanglementis still under intensive research. To understand the multipartite entanglement, the distributed entanglement hasbeen presented.(Derzhkoand et al, 2006) The residual entanglement is generalized to the multipartite entanglement.(Hill and et al, 1997) The multipartiteentanglement in Ising model is also studied.(Garate, Affleck.2010).

Dzyaloshinskii has shown(Dzyaloshinsky, 1958)that an antisymmetricexchange $\vec{D}_{ij}.(\vec{S}_i \times \vec{S}_j)$ should be consideredin these magnetic materials. Later, Moriya has shown[Moriya T.1960]that inclusion of spin orbit coupling on magnetic ions in1st and 2nd order leads to antisymmetric and anisotropicexchange respectively.This interaction is, however, rather difficult to handle analytically, but it is one of the agents responsible for magnetic frustration. Since this interaction may induce spiral spin arrangements in theground state(O'Connor, Wootters. 2001), it is closely involved with ferroelectricity in multiferroic spin chains(Osborne , Nielsen. 2002,Oshikawa, Affleck.1999). Besides, theDM interaction plays an important role in explainingthe electron spin resonance experiments in some one-dimensional antiferromagnets(Rojas, Rojas, Ananikian, de Souza.2012). Moreover, the DM interaction modifies the dynamic properties(Seki, Yamasaki, Soda, Matsuura, Hirota, Tokura.2008)and quantum entanglementof spin chains(Sudan, Luscher,Lauchli. 2009). Behaviors of quantum and classical correlations in spin chain with DM interaction also was discussed(Wootters,1998).The Hamiltonian of the Izing model with nearest and next-nearest neighbor interactions with added Dzyaloshinskii-Morriya interactionis given by:

$$H = \sum_i (J_1 S_i^z S_{i+1}^z + J_2 S_i^z S_{i+2}^z + \vec{D}.(\vec{S}_i \times \vec{S}_{i+1})) \quad (1)$$

whereis $\vec{S}_i$ spin-1/2 operator on the i-th site, and J_1,J_2 are the exchange constant.. We are able to get the thermal average of the two-qubit operator, calledthe reduced two-qubit density operator.Since these density operators are spatially separated, we could obtain the concurrence(entanglement) directly in the thermodynamic limit. The thermal entanglement (concurrence) is constructed for different valuesof the anisotropic Heisenberg parameter, DM interaction and temperature. In this paper we study the thermal entanglement of three qubits of Izing model with nearest and next-nearest neighbor interactions with added Dzyaloshinskii-Morriya interaction at low temperatureswhere $J_1 \rangle 0$ ($J_1 \langle 0$) is the nearest-neighbor (NN), and $J_2 \rangle 0$ ($J_2 \langle 0$)- the frustrating diagonal nextnearest- neighbor (NNN) coupling on a 1-D lattice.

Three-qubitthermal entanglement

Quantum entanglement is a special type of correlation,which only arises in quantum systems. Entanglement reflects nonlocal distributions between pairs of particles,even if they are removed and do not directly interactwith each other. In order to measure the entanglement of anisotropic Heisenberg qubits in the IsingHeisenberg model on a chain, we study the concurrence (entanglement) of the two qubits Heisenberg, which interacts with two nodal Ising spins using the definition proposed by Wooters et al. [Wooters,2009, Yu CS and et al. 2005] we focus on the entanglement of formation of three-qubit in two inequivalent,symmetric and non-symmetric pair wise entanglement ways.By labeling the 3-qubits as 1,2,3sequentially.The symmetric reduced density matrix ρ_{13} is defind as $\rho_{13} = \mathrm{tr}_2(\rho)$,where ρis the density matrix of 3-qubits.The non-symmetric reduced matrix is $\rho_{12} = \mathrm{tr}_3(\rho)$.Before present our results,we briefly review the definition of concurrence .Letρ_{ij} be density matrix of a pair of qubit i and j .The concurrence corresponding to density matrix is defind as

$$C_{ij} = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \quad (2)$$

Where the quantities $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ are square roots of the eigenvalue of the operator

$$\rho_{ij} = (\delta_y \otimes \delta_y)\rho_{ij}^*(\delta_y \otimes \delta_y) \quad (3)$$

The concurrence $C_{ij} = 0$ corresponds to an non-entangled state and $C_{ij} = 1$ corresponds to a maximally entanglement state.A straight forward calculation gives the eigenstates and the eigenvalues of 3-qubit of Eq (1).

Entanglement in three-qubitIsing model with (J_1_J_2) anisotropic Heisenberg parameter and DM interaction

The Hamiltonian of thethree qubits of the model is:

$$H = J_1(S_1^z S_2^z + S_2^z S_3^z) + J_2\,(S_1^z S_3^z) + D_z\left(S_1^x S_2^y - S_1^y S_2^x\right) + D\left(S_2^x S_3^y - S_2^y S_3^x\right) \quad (4)$$

The Hamiltonian S_1^z, S_2^z and S_3^z have z –component of spin particles, respectively the first, second and third, J_1,J_2 values of the anisotropic Heisenberg parameter and D ,Ratio of coupling between spin interaction in the Z direction. Hamiltonian matrix form as follows:

$$H = \frac{1}{4}\begin{bmatrix} 2J_1 + J_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -J_2 & 2iD & 0 & 0 & 0 & 0 & 0 \\ 0 & -2iD & -2J_1 + J_2 & 0 & 2iD & 0 & 0 & 0 \\ 0 & 0 & 0 & -J_2 & 0 & 2iD & 0 & 0 \\ 0 & 0 & -2iD & 0 & -J_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2iD & 0 & -2J_1 + J_2 & 2iD & 0 \\ 0 & 0 & 0 & 0 & 0 & -2iD & -J_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2J_1 + J_2 \end{bmatrix} \quad (5)$$

In what follows, we focus on thermal entanglement behavior of three qubits in the SY and NSYways. The thermal elements of the density matrix operator ρ(T) = exp(−βH), where Z = Tr[exp(−βH)] is thepartition function and β = 1/kT , for SY and NSY ways are presented in sequence. In the NSY case $\rho_{nsy}(T)$ is given as

$$\rho_{11}=\rho_{44}=\frac{1}{Z}\left[\frac{1}{2}e^{-\beta E_1}+e^{-\beta E_3}+\frac{1}{b^2+2}e^{-\beta E_5}+\frac{1}{a^2+2}e^{-\beta E_7}\right]$$
$$\rho_{22}=\rho_{33}=\frac{1}{Z}\left[\frac{1}{2}e^{-\beta E_1}+\frac{2b^2}{b^2+2}e^{-\beta E_5}+\frac{2a^2}{a^2+2}e^{-\beta E_7}\right]$$
$$\rho_{23}=(\rho_{32})*=\frac{-2i}{Z}\left[\frac{b}{b^2+2}e^{-\beta E_5}+\frac{a}{a^2+2}e^{-\beta E_7}\right](6)$$

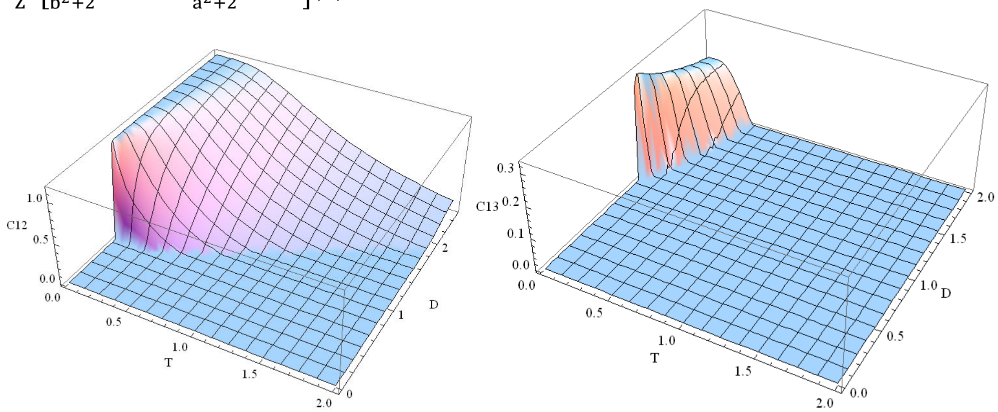

Figure 1.The thermal entanglement throughnon-symmetric way$c_{12}$and symmetric way c_{13} for antiferromagnetic case J1/ J2=1as function of DM and temperature

and in the SY case $\rho_{sy}(T)$ is written as

$$\rho_{11}=\rho_{44}=\frac{1}{Z}\left[e^{-\beta E_3}+\frac{b^2}{b^2+2}e^{-\beta E_5}+\frac{a^2}{a^2+2}e^{-\beta E_7}\right]$$
$$\rho_{22}=\rho_{33}=\frac{1}{Z}\left[e^{-\beta E_1}+\frac{2}{b^2+2}e^{-\beta E_5}+\frac{2}{a^2+2}e^{-\beta E_7}\right]$$
$$\rho_{23}=\rho_{32}=\frac{1}{Z}\left[e^{-\beta E_1}-\frac{2}{b^2+2}e^{-\beta E_5}-\frac{2}{a^2+2}e^{-\beta E_7}\right] \qquad (7)$$

where$Z=2\exp(J_2/4T)+4\exp(J_1/4T)\cosh\left(\sqrt{(J_1-J_2)^2+8D^2}/4T\right)+2\exp(-J_1/2T)\exp(-J_2/2T)$;a=$\frac{(J_1-J_2)+q}{2d}$, b=$\frac{(J_1-J_2)-q}{2d}$,q=$\sqrt{(J1-J2)^2+8D^2}$.The eigenvalues of Eq(1) are given as

$$E_1=E_2=-\frac{J_2}{4} \quad , \quad E_3=E_4=\frac{J_1}{2}+\frac{J_2}{4}$$
$$E_5=E_6=-\frac{1}{4}(J_1+q) \quad , \quad E_7=E_8=-\frac{1}{4}(J_1-q)(8)$$

Figure 1.show symmetricC_{13},and non-symmetricC_{12},concurrences.In the both way is clear that in the absence of DM interaction the antiferromagnetic case is entanglement. Entanglement starts to increasing DM interaction and shows a competitive behavior between DM and T,which entanglement shows decreasing by increasing T.In antiferromagnetic chain, through non-symmetric case,is entangled.Entanglement starts to increasingfrom zero as soon as turn on DM and reaches its saturation value around D_c=1.

CONCLUSION

To summarize, we have investigated the effect of a Dzyaloshinsky-Moriya (DM) interaction on the thermal entanglement of Ising spin-1/2 model with nearest and next nearest neighbor interaction through the SY and the

NSY ways. We have calculated the concurrence through the SY and the NSY ways for AF cases. we illustrate the density plot of concurrence C as a function of T and DM interaction for a fixed value of J1/ J2=1. The density plot of concurrence (C = 1)the maximum entangled region, while (C = 0) is the unentangled region and middle of plot means the different degrees of entanglement (0 <C < 1). After that we use this representationfor the concurrence C, depending on the parameters ofthe Hamiltonian (1). The entangled region representedby different intensities depends also on the temperature. In Fig (2,3)it is clear For high temperature the fuzzy region increases,while for the low-temperature phase between the entangled region and the unentangled region the boundary becomes sharper.

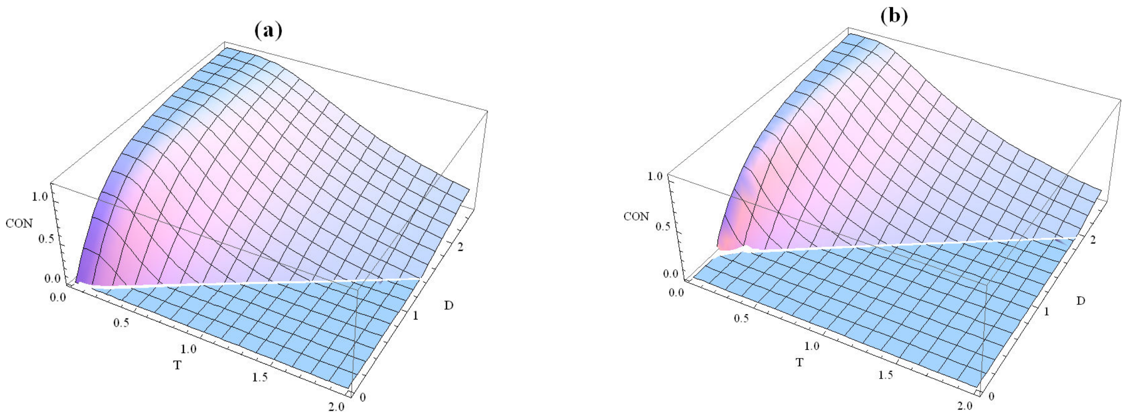

Figure 2.The concurrence is plotted through non-symmetric wayc_{12} for (a) J1=1, J2=-1 and (b) J1=-1, J2=1 as function of DM vector D and temperature

Usually the entangled region vanishes when the temperature increases; in some cases [see for instance Fig1.the entanglement vanishes asymptotically when the DM interaction is switched on.The entangled region is limited by the so-called threshold temperature, but for some other parameters [like inFig1] the threshold temperature onlyoccurs in the asymptotic limit. From our result we conclude that there is a peak in the concurrence as afunction of temperature and DM interaction.Our calculations show that the DM interaction. At the critical point D_c = 1.0, quantum entanglement are induced in the both SY and NSY ways.

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