Mining method selection by using an integrated model

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ABSTRACT: The problem of mining method selection is one of the most important decisions that should be made by mining managers and engineers. Selecting a proper underground mining method to accomplish extraction from a mineral deposit is very significant in terms of the economics, safety and the productivity of mining operations. The aim objective of this paper is to develop an integrated model to selection the best mining method by using effective criteria and at the same time, taking subjective judgments of decision makers into account. Proposed model is based on fuzzy analytic hierarchy process (FAHP) methodology and technique for order preference by similarity to ideal solution (TOPSIS). FAHP is applied to determine the weights of the evaluation criteria for mining method selection that these weights are inserted to the TOPSIS technique to rank the alternatives and select the most appropriate alternative. The proposed method is applied for Angouran Mine in Iran and finally the optimum mining methods for this mine are ranked. The study was followed by the sensitivity analysis of the results.

Keywords: Mining method selection, MCDM, FAHP, TOPSIS

INTRODUCTION

Selection of mining method is one of the most crucial decisions in the design stage of mine that mining engineers have to make. Selecting a mining method for mineral resources is completely dependent on the uncertain geometrical and geological characteristic of the resource (Azadeh et al, 2010). It is necessary to the unique characteristics of each mineral resource be taken into account in order to select the suitable mining method for the extraction of a certain resource, so that the utilized method would have the maximum technical-operational congruence with the geological and geometrical conditions of the mineral resource. To make the right decision on mining method selection, all effective criteria related to the problem should be taken into account. Increasing the number of the evaluation criteria in decision making problem makes the problem more complex, but also the rightness of the decision increases. Therefore, there is a need for alternative methods, which can consider all known criteria related to underground mining method selection in the decision making process (Alpay, Yavuz, 2009). The sensitivity of this decision has led to different solutions introduced by different researchers (Boshkov et al, 1973; Morrison, 1976; Laubscher, 1981; Nicholas, 1981; Hartman, 1982;

In the above-mentioned studies, mining method selection procedure has been looked from qualitative viewpoint (Azadeh et al, 2010). Likewise two of the problems of these approaches is lack of having correct relation between represent classes of parameters especially in near the boundary conditions and having the same relevance of all evaluation criteria. Therefore, these studies were neither enough nor complete, as it is not possible to design a methodology that will automatically choose a mining method for the ore body studies (Bitarafan, Ataei, 2004).

The merit of using multi-criteria decision making (MCDM) methods is their ability to solve complex and multi criteria problems by handling both quantitative and qualitative criteria. The MCDM methods are strong tools for determining the best alternative among a pool of the feasible alternatives. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is one common MCDM method that takes into consider the ideal and the anti-ideal solutions simultaneously. This technique is applied by different researches because of being rational, simple computations, and results are obtained in shorter time than other methods such as AHP (analytical hierarchy process) and ANP (analytic network process) (Fouladgar et al, 2011; Lashgari et al, 2011).

On the other hand, AHP is widely used to calculate the weights of evaluation criteria. This method use pair-wise comparison for obtaining the relative weights of criteria. AHP is strongly connected to human judgment and pairwise comparisons in AHP may cause evaluator’s assessment bias which makes the comparison judgment matrix inconsistent (Aydogan, 2011). Therefore, fuzzy analytical hierarchy process (FAHP) is employed to solve the bias problem in AHP.

The main aim of this paper is to develop an integrated model based on FAHP and TOPSIS methods in order to evaluate mining methods and select the best alternative in the Anguran mine. TOPSIS is employed to select a mining method and the FAHP is applied to calculate criteria weights.

The rest of this paper is organized as follows. In section 2, a brief review of fuzzy theory is presented, including fuzzy sets, fuzzy numbers, and linguistic variables. Section 3 illustrates the FAHP methodology for calculating the relative weights of evaluation criteria. The procedure of the TOPSIS method is described in section 4. The proposed model is presented in section 5. Section 6 presents an empirical study of mining method selection. A sensitivity analysis is conducted in section 7. Finally, concluding remarks are discussed in section 8.

Fuzzy logic

Fuzzy logic, introduced by Zadeh (1965), is a powerful tool for facing with the existing uncertainty, imprecise knowledge, and less of information. Fuzzy numbers may be of almost any shape (though conventionally they are required to be convex and to have finite area), but frequently they will be triangular (piecewise linear), s-shape (piecewise quadratic) or normal (bell shaped) (Kelemenis et al. 2011).

A triangular fuzzy number (TFN) is defined as \( \tilde{A} = (a, b, c) \); which a, b, and c are crisp numbers and \( a \leq b \leq c \).

A fuzzy number is defined by its membership function whose values can be any number in the interval \([0, 1]\). Assume that TFNs start rising from zero at \( x = a \); reach a maximum of 1 at \( x = b \); and decline to zero at \( x = c \) as shown in Figure 1. Then the membership function \( \mu_{\tilde{A}}(x) \) of a TFN is given by

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0, & x < a \\
(\frac{x - a}{b - a}, & a \leq x < b \\
(\frac{x - b}{c - b}, & b \leq x < c \\
0, & x > c 
\end{cases}
\]
Let $\tilde{a} = (a_1, b_1, c_1)$ and $\tilde{b} = (a_2, b_2, c_2)$ be two TFNs then the vertex method is defined to compute the distance between them by Eq. (2):

$$d(\tilde{a}, \tilde{b}) = \frac{1}{\sqrt{3}} \left[ (a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 \right]$$

**Fuzzy AHP**

Analytical hierarchy process (AHP), introduced by Saaty (1980), is a popular MCDM method that decomposes a sophisticated problem into a hierarchy. The elements of hierarchy levels are compared in pairs to assess their relative preference with respect to each other at each level (Singh & Benyoucef, 2011). The AHP is widely employed for tackling multi-criteria decision making problems in real world applications. However, in many practical cases the human preference model is uncertain and evaluator might be reluctant or unable to assign crisp values to the comparison judgments (Chan & Kumar, 2007). The merit of using a fuzzy approach is to determine the relative importance of attributes using fuzzy numbers instead of precise numbers (Önüt, Soner, 2008; Sun, Lin, 2009; Sun, 2010; Kara, 2011). There are many fuzzy AHP methods proposed on the basis of the concepts of the fuzzy set theory and hierarchical structure.

In this study, we use Chang’s extent analysis method (Chang, 1996) due to its computational simplicity and effectiveness.

Let $X = \{x_1, x_2, ..., x_n\}$ be an object set and $U = \{u_1, u_2, ..., u_m\}$ be a goal set. According to the method of Chang’s extent analysis, each object is taken and extent analysis for each goal, $g_i$, is performed, respectively. Therefore, $m$ extent analysis values for each object can be obtained, with the following signs:

$$M_{g_1}^1, M_{g_2}^2, ..., M_{g_i}^n, i = 1, 2, ..., n.$$ Where all the $M_{g_i}^j$ ($j = 1, 2, ..., m$) are TFNs.

The procedure of Chang’s extent analysis is defined in the following steps:

Step 1- The value of fuzzy synthetic extent with respect to $i$th object is calculated as:

$$S_i = \sum_{j=1}^{m} M_{g_j}^j \otimes \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_j}^j \right]^{-1}$$
To obtain $\sum_{j=1}^{m} M_{gi}^j$, perform the fuzzy addition operation of $m$ extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{gi}^j = \left( \sum_{j=1}^{m} l_j^i, \sum_{j=1}^{m} m_j^i, \sum_{j=1}^{m} u_j^i \right)$$

And to obtain $\left[ \sum_{j=1}^{m} \sum_{i=1}^{n} M_{gi}^j \right]^{-1}$, perform the fuzzy addition operation of $M_{gi}^j (j = 1, 2, \ldots, m)$ values such that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j = \left( \sum_{i=1}^{n} l_i^j, \sum_{i=1}^{n} m_i^j, \sum_{i=1}^{n} u_i^j \right)$$

And then calculate the inverse of the vector in Eq. (6) such that

$$\left[ \sum_{n=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} = \left( \frac{1}{\sum_{n=1}^{n} u_i^j}, \frac{1}{\sum_{n=1}^{n} m_i^j}, \frac{1}{\sum_{n=1}^{n} l_i^j} \right)$$

Step 2- The degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is assigned as

$$V(M_2 \geq M_1) = \sup_{y \geq x} \left[ \min(\mu_{M_1}(x), \mu_{M_2}(y)) \right]$$

And can be equivalently expressed as follows:

$$V(M_2 \geq M_1) = \text{hgt}(M_1 \cap M_2) = \mu_{M_2}(d) = \begin{cases} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise} \end{cases}$$

Both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$ are needed to compare $M_1$ and $M_2$.

Step 3- The degree of possibility for a convex fuzzy number to be greater than $k$ convex fuzzy numbers $M_i (i=1, 2, \ldots, k)$ can be computed by

$$V(M \geq M_1, M_2, \ldots, M_k) = V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \ldots \text{ and } (M \geq M_k)]$$

$$= \min_{i=1, 2, \ldots, k} V(M \geq M_i)$$

Assume that

$$d'(A_i) = \min V(S_i \geq S_k)$$
For $k = 1, 2, \ldots, n; \ k \neq i$.

Then the weight vector is obtained by

$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T$

Where $A_i$ ($i=1, 2, \ldots, n$) are n elements.

Step 4- The normalized weight vectors are resulted through normalization

$W = (d(A_1), d(A_2), \ldots, d(A_n))^T$

Where $W$ is a non-fuzzy number.

**TOPSIS technique**

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was first introduced by Hwang and Yoon (1981). TOPSIS method is based on the concept that the most appropriate alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). PIS minimize the cost criteria and maximize the benefit criteria, whereas the NIS minimizes the benefit criteria and maximizes the cost criteria (Kelemenis et al. 2011). There have been plenty of studies related with the TOPSIS method in the literature (Parkan & Wu, 1999; Gamberini et al. 2006; Yu et al. 2009; Chen et al. 2009; Antucheviciene et al. 2010; Tupenaite et al. 2010; Chang et al. 2010). The major steps of the TOPSIS can be described as follows:

**Step 1. Construct the decision matrix.**

To calculate the performance of a set of alternatives on a given set of criteria, the decision matrix of $m \times n$ dimension is formed, which $m$ and $n$ are the number of alternatives and criteria respectively.

**Step 2. Calculate the normalized decision matrix.** The normalized value $r_{ij}$ is calculated as

$r_{ij} = f_{ij} / \sqrt{\sum_{j=1}^{m} f_{ij}^2}$, \hspace{1cm} j = 1, \ldots, m; \ i = 1, \ldots, n$

**Step 3. Calculate the weighted normalized decision matrix.**

We can compute the weighted normalized decision matrix by considering the relative importance of evaluation criteria as

$V = [v_{ij}]_{m \times n}$

and

$v_{ij} = r_{ij} \times w_j$

Where $W = \{w_j : j = 1, 2, \ldots, n\}$ normalized criteria weights.

**Step 4. Identify positive ideal ($A^*$) and negative ideal ($A^-$) solutions.** The positive –ideal solution and the negative-ideal solution are shown in Eqs. (15), (16).
\[ A^* = (v^*_1, v^*_2, v^*_3, ..., v^*_n) = \left\{ \max_{i} v_{ij}(i = 1, 2, ..., n) \right\} \]

(16)

\[ A^- = (v^-_1, v^-_2, v^-_3, ..., v^-_n) = \left\{ \min_{i} v_{ij}(i = 1, 2, ..., n) \right\} \]

Step 5. Calculate separation measures. The distance of each alternative from \( A^* \) and \( A^- \) can be currently calculated using Eqs. (17), (18).

\[ d^+_i = \sum_{j=1}^{n} d(v_{ij}, v^+_{ij}) \quad , i = 1, 2, ..., m \]

(17)

\[ d^-_i = \sum_{j=1}^{n} d(v_{ij}, v^-_{ij}) \quad , i = 1, 2, ..., m \]

(18)

Step 6. Calculate the similarities to ideal solution. This step solves the similarities to an ideal solution by Eq. (19).

\[ CC^*_i = \frac{d^-_i}{d^-_i + d^*_i} \]

(19)

Step 7. Rank preference order. Choose an alternative with maximum \( CC^*_i \) or rank alternatives according to \( CC^*_i \) in descending order.

The proposed model

The proposed model for evaluating the underground mining methods in Angouran mine, contained of FAHP and TOPSIS methods, comprises of three main steps: (1) determine the main and sub evaluation criteria; (2) calculate the relative weights of criteria by FAHP and (3) evaluate the possible alternatives by TOPSIS and finally select the optimum alternative among a pool of alternatives. Schematic diagram of the proposed model for mining method selection is depicted in Figure 2.

In the step 1, after defining the problem, the feasible mining methods for the extraction process of the ore are identified. Next, the effective criteria of possible alternatives are determined. In the final phase of the step 1, the decision hierarchy is structured such that the goal is in the first level, evaluation criteria are in the second level, sub-criteria are in the third level, and possible alternatives are on the last level. In the step 2, after constructing the decision hierarchy, the relative weights of the evaluation criteria are obtained by using the FAHP technique. Based on these evaluation criteria, the required data in order to form the pairwise comparison matrix are collected from expert's knowledge. In the step 3, the performance ratings of the feasible alternatives corresponding to the evaluation criteria are assigned by applying the UBC technique. Finally, TOPSIS is applied to evaluate the alternatives and select the best underground mining method among a pool of alternatives.

An empirical application

The purpose of the empirical application is to illustrate the use of the suggested method. Angouran Zn–Pb
Deposit is located in the western Zanjan province about 450 km northwest of Tehran (Figure 3a). This deposit is one of the major zinc producers in Iran, a country with approximately 11 million tons of zinc metal constituent. Angouran has 16 million tons of ore with a zinc concentration of 26% and a lead concentration of 6%¹. This deposit is close to the Urumieh-Dokhtar Magmatic Arc, which is situated within one of a number of metamorphic inlier complexes in the central Sanandaj-Sirjan Zone of the Zagros orogenic belt (Gilg et al., 2005). A metamorphic core complex surrounds the Angouran deposit, which comprises amphibolites, serpentinites, gneisses, micaschists, and various, mainly calcitic and rarely dolomitic marbles. Some of the geological specifications of the area are represented in Figure 3b.

¹www.turquoisepartners.com
Determine the main and sub-criteria

The evaluation criteria should be determined that cover the requirements connected with the mining method selection problem. According to the UBC technique, the proposed set of criteria consists of eleven parameters. The main characteristics of the parameters are Ore body thickness (C1), Ore body dip (C2), Ore body shape (C3), grade distribution (C4), Ore body depth (C5), Ore body RSS\(^2\) (C6), Footwall RSS (C7), Hanging wall RSS (C8), Ore body RMR\(^3\) (C9), Footwall RMR (C10), and Hanging wall RMR (C11).

As a result, these eleven criteria were employed in the process of the evaluation and decision hierarchy is established accordingly as depicted in Figure 4. The hierarchy of mining method selection can be divided into three levels: level 1 includes the main goal of the hierarchy, which is selection the most optimum mining method. The criteria are on the second level. Level 3 comprises the feasible alternatives determined by the decision maker team, including Block Caving (A1), Sublevel Stoping (A2), Sublevel Caving (A3), Cut & Fill (A4), Top Slicing (A5), and Square Set Stoping (A6).

The Angouran ore body is located in the crest of an open anticlinal structure within the metamorphic basement that plunges eastward at 10–20° (Gilg et al, 2005). This ore body is some 600 m long in

\(^2\)Rock Substance strength
\(^3\)Rock mass rating
Northern-Southern line and 200–400 m across. The ore body is delimited by two major NNW-SSE and NW-SE trending faults and a third NE-SW fault (Boni et al, 2007).

In Angouran mine, extraction of deposit has been started from near surface by open pit mining and it has continued to the level of 2880 meters. According to increasing the extraction depth and environmental requirements, mine is designed to transfer from open pit to underground mining. For this reason, underground mining method should be selected; so that, the evaluation criteria under consideration be satisfied.

**Figure 4. Decision hierarchy**

After building the hierarchical structure, we designed an AHP questionnaire format and arrange the pair-wise comparisons matrix. Firstly each decision maker individually carry out pair-wise comparison by using Saaty’s 1–9 scale (Saaty, 1980) as shown in Table 1. The consistency of the decision maker’s judgments during the evaluation phase is calculated by consistency ratio (CR) that could be defined as follows (Aguaron et al, 2003):

\[
CR = \frac{\lambda_{\text{max}} - n}{n - 1}
\]

where \(\lambda_{\text{max}}\) is the principal eigenvalue and \(n\) is the rank of judgment matrix. The closer the inconsistency ratio to zero, the greater the consistency (Torfi et al, 2010). The resulting CR values for our case study are smaller than the critical value of 0.1, this show that there is no evidence of inconsistency.

**Calculate the relative weights of criteria by FAHP**

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Table 1. Pair-wise comparison scale (Saaty, 1980)

<table>
<thead>
<tr>
<th>Option</th>
<th>Numerical value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal</td>
<td>1</td>
</tr>
<tr>
<td>Marginally strong</td>
<td>3</td>
</tr>
<tr>
<td>Strong</td>
<td>5</td>
</tr>
<tr>
<td>Very strong</td>
<td>7</td>
</tr>
<tr>
<td>Extremely strong</td>
<td>9</td>
</tr>
<tr>
<td>Intermediate values to reflect fuzzy inputs</td>
<td>2, 4, 6, 8</td>
</tr>
<tr>
<td>Reflecting dominance of second alternative compared with the first reciprocals</td>
<td></td>
</tr>
</tbody>
</table>

The importance weights of the criteria determined by fifteen decision-makers that are obtained through Eq. (21) are presented in Table 5.

\[ \tilde{x}_{ij} = (l_{ij}, m_{ij}, u_{ij}) \]

\[ l_{ij} = \min \{x_{ij}^k\}, m_{ij} = \frac{1}{k} \sum_{k=1}^{k} x_{ij}^k, u_{ij} = \max \{x_{ij}^k\} \]

Where \( \tilde{x}_{ij} \) is the fuzzy importance weights of each criterion that are determined by all experts, \( x_{ij} \) is the crisp weight of each criterion, \( k \) is the number of expert (here, \( k \) is equal to 15).

The results derived from the computations according to the final fuzzy matrices provided in Tables 2 are presented in Tables 3. The weight calculation details by using FAHP are given below.

Table 2. Pairwise comparison matrix

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>(1,1)</td>
<td>(0.14,0.24,0.5)</td>
<td>(0.33,0.98,2)</td>
<td>(0.25,0.76,1)</td>
<td>(0.33,0.98,3)</td>
<td>(0.25,1.04,2)</td>
<td>(0.25,0.66,2)</td>
<td>(0.25,0.73,3)</td>
<td>(0.25,0.62,2)</td>
<td>(1.1,82,3)</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>(2.4,16,7)</td>
<td>(1,1,1)</td>
<td>(5.6,23,8)</td>
<td>(4.6,43,9)</td>
<td>(4.5,34,8)</td>
<td>(4.5,78,9)</td>
<td>(5.6,87,9)</td>
<td>(4.6,12,8)</td>
<td>(5.7,13,9)</td>
<td>(3.4,76,8)</td>
<td>(5.7,54,9)</td>
</tr>
<tr>
<td>C3</td>
<td>(0.5,1,02,3,0)</td>
<td>(0.12,0.16,0.2)</td>
<td>(1,1,1)</td>
<td>(0.2,0.67,1)</td>
<td>(0.25,0.74,1)</td>
<td>(0.5,1.23,3)</td>
<td>(0.33,0.87,1)</td>
<td>(0.5,0.96,2)</td>
<td>(0.5,0.12,13)</td>
<td>(0.33,0.54,1)</td>
<td>(0.5,1.12,3)</td>
</tr>
<tr>
<td>C4</td>
<td>(0.5,1,32,4)</td>
<td>(0.11,0.16,0.25)</td>
<td>(0.5,1,49,5)</td>
<td>(1,1,1)</td>
<td>(0.14,0.36,0.5)</td>
<td>(0.33,0.63,1)</td>
<td>(0.2,0.46,1)</td>
<td>(0.33,0.65,2)</td>
<td>(0.33,1.11,2)</td>
<td>(0.2,0.35,1)</td>
<td>(0.5,1.34,3)</td>
</tr>
<tr>
<td>C5</td>
<td>(1,79,4)</td>
<td>(0.12,0.19,0.25)</td>
<td>(1,1,35,4)</td>
<td>(2,2,78,7)</td>
<td>(1,1,1)</td>
<td>(2,3,78,5)</td>
<td>(4,5,67,8)</td>
<td>(1,2,46,4)</td>
<td>(2,2,56,4)</td>
<td>(0.17,0,37,2)</td>
<td>(1,3,12,5)</td>
</tr>
<tr>
<td>C6</td>
<td>(0.33,1,02,3)</td>
<td>(0.11,0.17,0.25)</td>
<td>(0.33,0.81,2)</td>
<td>(1,1,59,3)</td>
<td>(0,2,26,0.5)</td>
<td>(1,1,1)</td>
<td>(3,4,78,7)</td>
<td>(0,33,0.89,2)</td>
<td>(1,2,23,4)</td>
<td>(0,14,0.27,0.5)</td>
<td>(2,3,67,5)</td>
</tr>
<tr>
<td>C7</td>
<td>(0.5,0.96,4)</td>
<td>(0.11,0.15,0.2)</td>
<td>(1,1,15,3)</td>
<td>(1,2,17,5)</td>
<td>(0,12,0.17,0.25)</td>
<td>(0,14,0.21,0,33)</td>
<td>(1,1,1)</td>
<td>(0,2,27,0.5)</td>
<td>(0,25,0.98,3)</td>
<td>(0,2,0,45,1)</td>
<td>(0,5,1,23,2)</td>
</tr>
<tr>
<td>C8</td>
<td>(0.5,1,79,4)</td>
<td>(0.12,0.16,0.25)</td>
<td>(0,5,1,04,2)</td>
<td>(0,5,1,54,3)</td>
<td>(0,25,0.41,1)</td>
<td>(0,5,1,12,3)</td>
<td>(2,3,7,5)</td>
<td>(1,1,1)</td>
<td>(0,5,2,31,4)</td>
<td>(0,25,0.78,2)</td>
<td>(0,5,2,45,4)</td>
</tr>
<tr>
<td>C9</td>
<td>(0.33,1,37,5)</td>
<td>(0.11,0.14,0.2)</td>
<td>(0,33,0,83,2)</td>
<td>(0,5,0,90,3)</td>
<td>(0,25,0.39,0.5)</td>
<td>(0,25,0.45,1)</td>
<td>(0,33,1,02,4)</td>
<td>(0,25,0.43,2)</td>
<td>(1,1,1)</td>
<td>(0,33,0,91,3)</td>
<td>(1,2,31,4)</td>
</tr>
<tr>
<td>C10</td>
<td>(0.5,1,59,4)</td>
<td>(0.12,0.21,0.33)</td>
<td>(1,1,85,3)</td>
<td>(1,2,86,5)</td>
<td>(0,5,2,7,6)</td>
<td>(2,3,7,7)</td>
<td>(1,2,22,5)</td>
<td>(0,5,1,28,4)</td>
<td>(0,33,1,09,3)</td>
<td>(1,1,1)</td>
<td>(0,5,1,49,3)</td>
</tr>
<tr>
<td>C11</td>
<td>(0.33,0,55,1)</td>
<td>(0.11,0,13,0,2)</td>
<td>(0,33,0,89,2)</td>
<td>(0,33,0,75,2)</td>
<td>(0,2,0,32,1)</td>
<td>(0,2,0,27,0,5)</td>
<td>(0,5,0,81,2)</td>
<td>(0,25,0,41,2)</td>
<td>(0,25,0,43,1)</td>
<td>(0,33,0,69,2)</td>
<td>(1,1,1)</td>
</tr>
</tbody>
</table>

The value of fuzzy synthetic extent with respect to the ith object is calculated as

\[ S_1 = (4.26,9.3,21.5) \otimes (0.003,0.005,0.009) = (0.012,0.048,0.198) \]

\[ S_2 = (42,61.37,85) \otimes (0.003,0.005,0.009) = (0.12,0.32,0.783) \]
$S_3 = (4.74, 9.52, 20.23) \otimes (0.003, 0.005, 0.009) = (0.013, 0.05, 0.186)$

$S_4 = (4.15, 8.86, 20.75) \otimes (0.003, 0.005, 0.009) = (0.012, 0.046, 0.191)$

$S_5 = (15.29, 25.06, 44.25) \otimes (0.003, 0.005, 0.009) = (0.044, 0.131, 0.408)$

$S_6 = (9.45, 16.69, 28.25) \otimes (0.003, 0.005, 0.009) = (0.027, 0.087, 0.26)$

$S_7 = (5.03, 8.74, 20.28) \otimes (0.003, 0.005, 0.009) = (0.014, 0.046, 0.187)$

$S_8 = (6.62, 16.3, 29.25) \otimes (0.003, 0.005, 0.009) = (0.019, 0.085, 0.269)$

$S_9 = (4.69, 9.74, 25.7) \otimes (0.003, 0.005, 0.009) = (0.013, 0.051, 0.237)$

$S_{10} = (8.45, 19.96, 41.33) \otimes (0.003, 0.005, 0.009) = (0.024, 0.104, 0.381)$

$S_{11} = (3.84, 6.25, 14.7) \otimes (0.003, 0.005, 0.009) = (0.011, 0.033, 0.135)$

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
<th>$S_6$</th>
<th>$S_7$</th>
<th>$S_8$</th>
<th>$S_9$</th>
<th>$S_{10}$</th>
<th>$S_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V(S_1 \geq \cdots)$</td>
<td>0.224</td>
<td>0.994</td>
<td>1</td>
<td>0.653</td>
<td>0.816</td>
<td>1</td>
<td>0.83</td>
<td>0.987</td>
<td>0.758</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_2 \geq \cdots)$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_3 \geq \cdots)$</td>
<td>1</td>
<td>0.198</td>
<td>1</td>
<td>0.638</td>
<td>0.81</td>
<td>1</td>
<td>0.826</td>
<td>0.993</td>
<td>0.75</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_4 \geq \cdots)$</td>
<td>0.987</td>
<td>0.207</td>
<td>0.981</td>
<td>0.636</td>
<td>0.801</td>
<td>1</td>
<td>0.816</td>
<td>0.975</td>
<td>0.74</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_5 \geq \cdots)$</td>
<td>1</td>
<td>0.604</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_6 \geq \cdots)$</td>
<td>1</td>
<td>0.377</td>
<td>1</td>
<td>1</td>
<td>0.833</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.93</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_7 \geq \cdots)$</td>
<td>0.98</td>
<td>0.197</td>
<td>0.977</td>
<td>0.997</td>
<td>0.628</td>
<td>0.794</td>
<td>0.81</td>
<td>0.971</td>
<td>0.736</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_8 \geq \cdots)$</td>
<td>1</td>
<td>0.39</td>
<td>1</td>
<td>1</td>
<td>0.832</td>
<td>0.992</td>
<td>1</td>
<td>1</td>
<td>0.928</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_9 \geq \cdots)$</td>
<td>1</td>
<td>0.303</td>
<td>1</td>
<td>1</td>
<td>0.708</td>
<td>0.85</td>
<td>1</td>
<td>0.864</td>
<td>0.8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_{10} \geq \cdots)$</td>
<td>1</td>
<td>0.548</td>
<td>1</td>
<td>1</td>
<td>0.927</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$V(S_{11} \geq \cdots)$</td>
<td>0.886</td>
<td>0.052</td>
<td>0.878</td>
<td>0.901</td>
<td>0.484</td>
<td>0.67</td>
<td>0.903</td>
<td>0.69</td>
<td>0.87</td>
<td>0.609</td>
<td></td>
</tr>
</tbody>
</table>
Then priority weights are computed by using Eq. (9):

\[
d' = \min(\text{values})
\]

\[
d'(C1) = \min(0.224, 0.994, 1, 0.653, 0.816, 1, 0.83, 0.987, 0.758, 1) = 0.224
\]
\[
d'(C2) = \min(1, 1, 1, 1, 1, 1, 1, 1, 1, 1) = 1
\]
\[
d'(C3) = \min(0.198, 1, 0.638, 0.811, 1, 0.826, 0.993, 0.751, 1) = 0.198
\]
\[
d'(C4) = \min(0.987, 0.207, 0.981, 0.636, 0.801, 1, 0.816, 0.975, 0.741, 1) = 0.207
\]
\[
d'(C5) = \min(1, 0.604, 1, 1, 1, 1, 1, 1) = 0.604
\]
\[
d'(C6) = \min(1, 0.377, 1, 1, 0.833, 1, 1, 1, 0.931) = 0.377
\]
\[
d'(C7) = \min(0.98, 0.197, 0.977, 0.997, 0.628, 0.794, 0.81, 0.971, 0.736, 1) = 0.197
\]
\[
d'(C8) = \min(1, 0.39, 1, 1, 0.832, 0.992, 1, 1, 0.928, 1) = 0.39
\]
\[
d'(C9) = \min(1, 0.303, 1, 1, 0.708, 0.851, 0.864, 0.81) = 0.303
\]
\[
d'(C10) = \min(1, 0.548, 1, 1, 0.927, 1, 1, 1, 1) = 0.548
\]
\[
d'(C11) = \min(0.886, 0.052, 0.878, 0.901, 0.484, 0.67, 0.903, 0.69, 0.87, 0.609) = 0.052
\]

Priority weights form \(W' = (0.224, 1, 0.198, 0.207, 0.604, 0.377, 0.197, 0.39, 0.303, 0.548, 0.052)\) vector. After the normalization of these values priority weights respect to main goal are calculated as (0.055, 0.244, 0.048, 0.051, 0.147, 0.092, 0.048, 0.095, 0.074, 0.134, 0.013). Mentioned priority weights have indicated for each criterion in Table 4. The results of the FAHP analysis for relative weights of the evaluation criteria are summarized in Figure 5.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Local weights</th>
<th>Global weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.224</td>
<td>0.055</td>
</tr>
<tr>
<td>C2</td>
<td>1</td>
<td>0.244</td>
</tr>
<tr>
<td>C3</td>
<td>0.198</td>
<td>0.048</td>
</tr>
<tr>
<td>C4</td>
<td>0.207</td>
<td>0.051</td>
</tr>
<tr>
<td>C5</td>
<td>0.604</td>
<td>0.147</td>
</tr>
<tr>
<td>C6</td>
<td>0.377</td>
<td>0.092</td>
</tr>
<tr>
<td>C7</td>
<td>0.197</td>
<td>0.048</td>
</tr>
<tr>
<td>C8</td>
<td>0.39</td>
<td>0.095</td>
</tr>
<tr>
<td>C9</td>
<td>0.303</td>
<td>0.074</td>
</tr>
<tr>
<td>C10</td>
<td>0.548</td>
<td>0.134</td>
</tr>
<tr>
<td>C11</td>
<td>0.052</td>
<td>0.013</td>
</tr>
</tbody>
</table>
Determine the final rank and select the best alternative by TOPSIS

In this step, evaluation matrices are established by fifteen decision makers for evaluating the underground mining methods under different criteria based on the UBC technique. In this study, all criteria are benefit criteria; thus the higher the score, the better the performance of the mining method is. The performance ratings of mining methods with respect to each criterion are determined by using the UBC technique and the results are presented in Table 5.

<table>
<thead>
<tr>
<th>Table 5. Performance ratings of alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>A1</td>
</tr>
<tr>
<td>A2</td>
</tr>
<tr>
<td>A3</td>
</tr>
<tr>
<td>A4</td>
</tr>
<tr>
<td>A5</td>
</tr>
<tr>
<td>A6</td>
</tr>
</tbody>
</table>

After forming the weighted decision matrix, the positive-ideal solution (PIS, A*) and the negative-ideal solution (NIS, A-) are derived. Finally, alternatives are ranked in descending order as presented in Table 7. According to CC values, the ranking of the alternatives in descending order are A4, A2, A3, A1, A6, and A5. The proposed model indicates that Cut & Fill (A4) is the best method with CC value of 0.819. Rankings of the alternatives according to CC values are depicted in Figure 6.

After forming the evaluation matrix, the second phase is to calculate the normalized decision matrix by using Eq. (14). Next, using the criteria weights obtained by FAHP, the weighted decision matrix is derived as presented in Table 6.

<table>
<thead>
<tr>
<th>Table 6. Weighted decision matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>A1</td>
</tr>
<tr>
<td>A2</td>
</tr>
<tr>
<td>A3</td>
</tr>
<tr>
<td>A4</td>
</tr>
<tr>
<td>A5</td>
</tr>
<tr>
<td>A6</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 7. TOPSIS results</th>
</tr>
</thead>
<tbody>
<tr>
<td>d_i+</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>A1</td>
</tr>
<tr>
<td>A2</td>
</tr>
<tr>
<td>A3</td>
</tr>
<tr>
<td>A4</td>
</tr>
<tr>
<td>A5</td>
</tr>
<tr>
<td>A6</td>
</tr>
</tbody>
</table>
Sensitivity analysis

In order to identify the cause of the difference in the outcome of the proposed model, a sensitivity analysis is conducted. This technique generates different scenarios that may change the priority of alternatives and be needed to reach a consensus. If the ranking order be changed by increasing or decreasing the importance of the criteria, the results are expressed to be sensitive otherwise it is robust. In this study, sensitivity analysis is implemented to see how sensitive the alternatives change with the importance of the criteria. This tool graphical exposes the importance of criteria weights in selecting the optimal alternative among the feasible alternatives. The main goal of sensitivity analysis is to see which criteria is most significant in influencing the decision making process. For this reason, eleven experiments were conducted that each experiment is generated by an increase of 20% in the amount of the weight of the criterion under consideration. It can be seen from Fig. 7 that alternative A4 has the highest score in all experiments. Therefore, it can be resulted that the decision making process is not sensitive to the criteria weight with alternative A4 (Cut & Fill) emerging as the winner.

CONCLUSION

The process of mining method selection is a methodology for evaluating the proper alternatives and selecting the best alternative with respect to criteria under consideration. The main goal of this study is to evaluate the feasible alternatives and select the most appropriate candidate among a pool of alternatives by
using the MCDM methods. According to the complex structure of the problem, inaccurate and imprecise data, less of information, and inherent uncertainty, the usage of the fuzzy sets can be useful.

In this paper, an integrated model based on FAHP and TOPSIS is developed. FAHP based on the extent analysis technique is applied to obtain weights of the evaluation criteria, while TOPSIS is utilized to prioritize the feasible alternatives. The weights derived from FAHP are involved in the problem of the mining method selection by using them in TOPSIS calculations and ranking order is determined based on these weights. Finally, the alternative with the highest score is selected. Also, sensitivity analysis was conducted to determine the influence of criteria weights on the problem of the mining method selection. The strength of the proposed model is the ability to evaluate and rank alternatives under partial or lack of quantitative information.

ACKNOWLEDGEMENT

The authors would like to acknowledge the financial support of Ayerma International Industrial and Mining Research Company for this research.

REFERENCES


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