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# Exertion Approximation WKB on Specific Quantity Problem and Analysis of Results Compare With Results of Numeral Methods

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**ABSTRACT:** Elastic material's changes in shape under different pressures cause specific amount problems. This paper is concerned solving these problems with regard to asymptotic methods. Numeral methods are useful until vacillations are normal. But when vacillations are high numeral methods are not so useful. By application of asymptotic methods obtained results are more precise and clear. In this research the main purpose is to apply WKB method. For solving equations in elastic pages and compare with numeral results of [5].the researcher at first will go through analyzing the curved and bending elastic pages which are made of Neo-Hookean material and then by using WKB according to [4, 5, 6, 8] will solve specific amount problems. In fact, by using this method, dependence of $\theta_0$ or $\lambda$ to big parameter $\mu = n\pi \gg 1$ will be shown. From physical aspects, the purpose of analyzing curve is finding some amount of $\lambda\ (r_1)$ or $\lambda(r_2)$ while page is broken during bending. Because of huge amount accounts which are not possible by hand, Mathematics Software for solving equations in all stages will be used. This software is powerful software in solving complex curves in mathematics, drawing diagrams and solving differential equations, especially when WKB method is used. Other mathematics software don't have such usability.
***Keywords****:* WKB systematic method, Specific amount problems, asymptotic results

## INTRODUCTION

Research on figures change of elastic structures which are under pressure is an important issue For example for transferring burnable material such as oil from marine ways. Research on rate of pressure on pipes which carry this material is very important. But many equations in physical and mechanical problems have specific situations and finding absolute results is very difficult. For example when a big or small parameter is shown in equation, there are asymptotic extensions for solving equations which are model of WKB approximate asymptotic extensions. When a small parameter is appear in equation, this estimate is used. It should be mentioned that three positions in WKB asymptotic analyses is regarded:

1)m>>1, $\zeta$ = O(1),

2) m = O(1), $\zeta$ =>>1,

3) m>>1, $\zeta$=O(m).

Asymptotic methods, especially WKB are used extensively for solving stable problems. However few researches for bending elastic pages used form this method. In this research, with WKB analyses the equations obtained elastic pages will be solved and then analyzing results with Haughton researches results will be done. Of course the results of WKB method are obtained with regarding third position of asymptotic analyses.

### Statement of problem

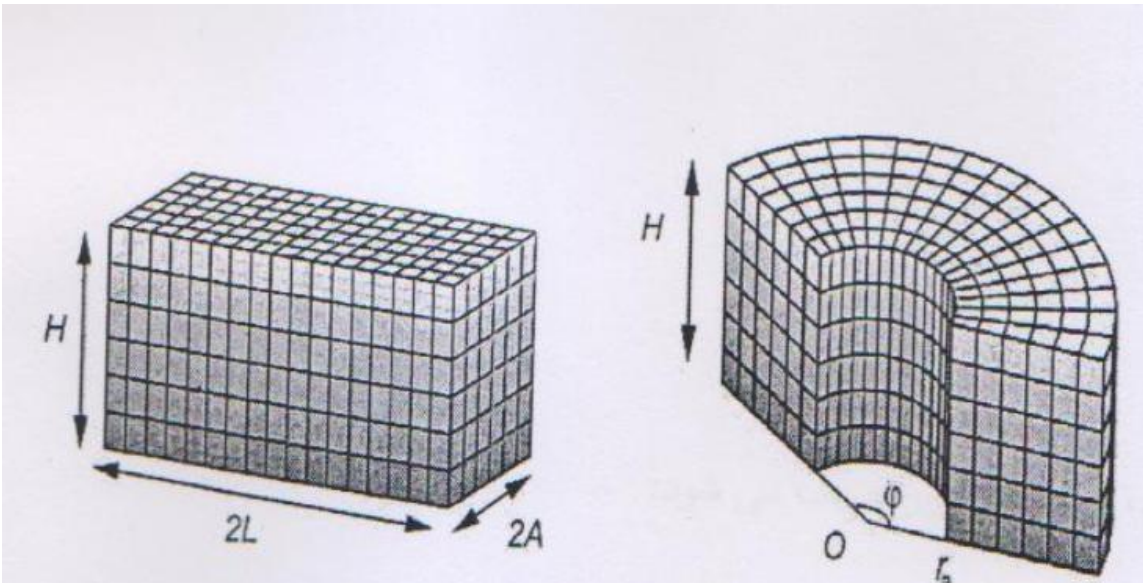
Let's consider elastic plate which before changing shapes is located in this area:

$$B_o(X_1, X_2, X_3) = \{(X_1, X_2, X_3)\epsilon R^3 | -A \le X_1 \le A, -L \le X_2 \le L, 0 \le X_3 \le H\}, \qquad (1\text{-}1)$$

That $X_1, X_2, X_3$ is Decartesian coordinates of homogenous, isotope three aspects' elastic plate. Elastic plate after changing shape according to polar coordinates is located.

$$B_e(r,\theta,z) = \{r_1 \le r \le r_2, -\theta_0 \le \theta \le \theta_0, 0 \le z \le H\lambda_3\} \quad (2\text{-}2)$$

That $\lambda_3$ is main extension.




Figure(1).A elastic plate before& after deformation

After shape's changing $r_1, r_2$ are internal and external radius, H plate height and $\theta_0$ is bending angel. But because page cannot pull back into itself, there is need to be so: $0 < \theta_0 = \alpha L \le \pi$.

Shape's changing in equations is as following:

$$r = \sqrt{d + \frac{2X_1}{\alpha\lambda_3}} \quad \theta = \alpha X_2, \quad z = X_3,$$

Fixed d in this relation is as following:

$$d = \frac{(4\alpha^2 A^2 + 1)^{\frac{1}{2}}}{\alpha^2 \lambda_3},$$

If A is regarded as $(A = 1)$ , $r_1, r_2$ will result in:

$$r_1 = \sqrt{d + \frac{2}{\alpha\lambda_3}}, \qquad r_2 = \sqrt{d - \frac{2}{\alpha\lambda_3}} .$$

Changing shape of Tensor Gradian will be defined as:

$$\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

That $\lambda_1, \lambda_2, \lambda_3$ are main extensions.

$\lambda_1 = \lambda^{-1} \lambda_3{}^{-1}, \quad \lambda_2 = \lambda, \quad \lambda = \alpha r$

The relation ( $\hat{n}\, T^n = \overline{\sigma}$ ) is well known as Kochi relation and shows tension vector's relation in arbitrary point $\hat{n}$ with tension Tensor $\overline{\sigma}$ in noted point.

$$\overline{\sigma}_{ii} = \overline{\sigma}_i - p = \lambda_i \frac{\partial w}{\partial \lambda_i} - p, \qquad i=1, 2, 3$$

Here P is pressure and $w_i = \frac{\partial w}{\partial \lambda_i}$ ,

Homage energy dependent which is indicative of stored energy in material is outcome of changing shape for Neo-Hookean according [5] is as following:

$$W(\lambda_1, \lambda_2, \lambda_3) = 2\,\tau \left(\lambda_1{}^2 + \lambda_2{}^2 + \lambda_3{}^2 - 3\right)$$

In this $\tau$ is module cutting of elastic material.

**Dominant Equations**

According to [5] balance equation for elastic page made of Neo–Hookean in absence of gathered pressures is as following:

$\mathrm{Div}\,\dot{\chi} = 0, \quad \mathrm{Div}\dot{\mathrm{x}} = 0,$

In that Div is durejanse operator and $\dot{\mathrm{x}}$ is tension vector and $\dot{\chi}$ is tension tansor (after changing shape) ,that is shown as:

:

$\dot{\chi} = B\bar{F} + p\bar{F}^T - p^* I,$

In that B is fourth tensor in present situation and $\bar{F}$is degree of changing place in present situation , P pressure in situation with limited tension , $p^*$ difference in pressure ( before and after changing shape) , I is single matrix . Now $\bar{F}$ is written as matrix:

$$\bar{F} = \begin{bmatrix} u_r & \frac{1}{r}(u_r - v) & u_z \\ v_r & \frac{1}{r}(u + v_\theta) & v_z \\ w_r & \frac{1}{r} w_\theta & w_z \end{bmatrix},$$

Because material is without density $\mathrm{tr}(\bar{F}) = 0$ so the result is this:

$$u_r + \frac{1}{r}(u + v_\theta) + w_z = 0,$$

With replacing and then after simplifying is as following:

$$p_r^* = B_{1111} u_{rr} + B_{2121} \frac{u_{\theta\theta}}{r^2} + B_{3131} u_{zz} + (rB_{1122} - B_{2121} + B_{1221}) \frac{u_r}{r} + (rB'_{1221} - B_{2222}) \frac{u}{r^2} + (rB'_{1122} - B_{2121} - B_{2222}) \frac{v_\theta}{r^2} + (B_{1331}) w_{rz} + (B_{1122} + B_{1221}) \frac{v_{r\theta}}{r} + rB'_{1133} + B_{1133} - B_{2233}) \frac{w_z}{r},$$

$$p_\theta^* = (rB'_{1221} + B_{1122} - B_{1111} + B_{1221} + B_{2222}) \frac{u_\theta}{r} + (B_{1122} + B_{1221}) u_{r\theta} + (rB_{1212}) v_{rr} + (rB'_{1212} + B_{1212})\, v_r - (rB'_{1212} + B_{1122} + B_{1221} - B_{1111}) \frac{v}{r} + B_{2222} \frac{v_{\theta\theta}}{r} + rB_{3232} v_{zz} + (B_{2233} + B_{3223}) w_{\theta z},$$

$$p_z^* = (rB'_{1331} + rp' + B_{2233} + B_{1331}) \frac{u_z}{r} + (B_{1331} + B_{1133}) u_{rz} + (B_{2332} + B_{2233}) \frac{v_{\theta z}}{r} + B_{1313} w_{rr} + (rB'_{1313} + B_{1313}) \frac{w_r}{r} + B_{3333} w_{zz} + B_{2323} \frac{w_{\theta\theta}}{r^2}.$$

In these relations $r\,p'$ is as following

$$r\,p' = B_{1122} + B_{1221} - B_{1111} - B_{2121},$$

because we want individual answers in regard to $r, \theta$ so we consider replacing dependent as following:

$$\begin{cases} u = f(r) \cos m\,\theta \cos \xi\, z, \\ v = g(r) \sin m\,\theta \cos \xi\, z, \\ w = h(r) \cos m\,\theta\ \sin \xi\, z, \\ p^* = k(r)\ \cos m\,\theta\ \cos \xi\, z. \end{cases}$$

$\xi$ is mode (depend to height) that with help of situations is specified and if we want to tension be zero at the two end of page, $\xi$ should take these quantities :

$$\xi = \frac{k\,\pi}{H\,\lambda_3}, \qquad (k = 1,2,3)$$

And $m$ which is called angle mode is obtained from this relation

$$m = \frac{n\,\pi}{\alpha\,L},$$

And also this can be written as $m = \frac{\mu}{\theta_0}$. $(\mu = n\,\pi)$

Now ,because tension at the two end of page is zero , border situations is as following:

$$\begin{cases} 2\,\lambda_1^2\, f'(r) - k(r) = 0, \\ r\,g'(r) - g(r) - m\,f(r) = 0, \\ h'(r) - \xi\ f(r) = 0. \end{cases}$$

So, finally we have:

$$\alpha=\frac{1-\lambda^4}{4\,\lambda^2},\ \ \mu=n\,\pi,\ \ \theta_0=\alpha\,L,\ \ \lambda=\alpha\,r.$$

And all these relations are apply for obtaining specific amount problem.

**Exertion Approximation WKB On Specific Quantity Problem**

For this specific amount problem, answer of WKB is considered as so:

$$\begin{Bmatrix} f \\ g \\ k \end{Bmatrix}=\begin{Bmatrix} F \\ G \\ m\,K \end{Bmatrix} exp(m\ \int^r s(x)dx\ \ )$$

That in which $F,G,K$ are dependedts of $r$ and $O(1)$ rank .

These $F,G,K$ can be extend to:

$F=F_0+\frac{F_1}{m}+\frac{F_2}{m^2}+\cdots,$

$G=G_0+\frac{G_1}{m}+\frac{G_2}{m^2}+\cdots,$

$K=m\,K_0+K_1+\frac{K_2}{m}+\cdots,$

Our goal is to obtain six independent answers for $F,G,K$ . So we use from this symbol $\begin{cases} F^{(i)} \\ G^{(i)} \\ K^{(i)} \end{cases}$ that (i=1,2,..6).

and $F_0,F_1,F_2,G_0,G_1,G_2,K_0,K_1,K_2,\ldots$ are dependents of $r$ that should be specified.

Now with replacing $F^{(i)},G^{(i)},K^{(i)}$ in equations and obtaining solving situations for $F_0^{(i)},G_0^{(i)},K_0^{(i)},F_1^{(i)},G_1^{(i)},K_1^{(i)},F_2^{(i)},G_2^{(i)},K_2^{(i)}$ and for repeated and unrepeated radicals, equations are simplified and solved. Finally after obtaining$F^{(i)},G^{(i)},K^{(i)}$, public answers are defined so:

$f=\sum_{i=1}^{6} c_i\ \ F^{(i)}E^{(i)}(r),$

$g=\sum_{i=1}^{6} c_i\ \ G^{(i)}E^{(i)}(r),$

$k=\sum_{i=1}^{6} c_i\ \ K^{(i)}E^{(i)}(r).$

On that $E^{(i)}(r)=exp(m\ \int s^{(i)}(x)dx)$ and we specify amount of $c_i$ s with placing $f,g,k$ in border situations. With replacing we will have:

$\sum_{i=1}^{6} c_i\ \ \alpha^{(i)}E^{(i)}(r)=0,$

$\sum_{i=1}^{6} c_i\ \ \beta^{(i)}E^{(i)}(r)=0,$

$\sum_{i=1}^{6} c_i\ \ \gamma^{(i)}E^{(i)}(r)=0.$

As:

$\alpha^{(i)}=m\,\alpha_1^{(i)}+\alpha_2^{(i)}+\frac{1}{m}\ \alpha_3^{(i)}+O\left(\frac{1}{m^2}\right),$

$\beta^{(i)}=m\,\beta_1^{(i)}+\beta_2^{(i)}+\frac{1}{m}\ \beta_3^{(i)}+O\left(\frac{1}{m^2}\right),$

$\gamma^{(i)}=m^2\,\gamma_1^{(i)}+m\,\gamma_2^{(i)}+\gamma_3^{(i)}+O\left(\frac{1}{m}\right).$

We can explain above relations as following:

$\sum_{j=1}^{6} M_{ij}\,c_j=0,\qquad (i=1,2,\ldots,6)$

For getting one unice answer for $c_i$ , M matrix Determine should be zero

$detM=0,$

With using [4] reference, we can decrease matrix determine $Mc=0$ into following form:

$$\begin{bmatrix} \alpha^1(r_2) & \alpha^3(r_2) & \alpha^5(r_2) \\ \beta^1(r_2) & \beta^3(r_2) & \beta^5(r_2) \\ \gamma^1(r_2) & \gamma^3(r_2) & \gamma^5(r_2) \end{bmatrix}=0,$$

$$\begin{bmatrix} \alpha^2(r_1) & \alpha^4(r_1) & \alpha^6(r_1) \\ \beta^2(r_1) & \beta^4(r_1) & \beta^6(r_1) \\ \gamma^2(r_1) & \gamma^4(r_1) & \gamma^6(r_1) \end{bmatrix}=0.$$

In this paper, we only discuss about$r=r_1$. So with placing second matrix equal zero we will have:

$H_1(\lambda)+\frac{1}{\mu}\ H_2(\lambda)+\frac{1}{\mu^2}\ H_3(\lambda)+\cdots=0,$

On that:

$\lambda = c_0 + \frac{1}{\mu} c_1 + \frac{1}{\mu^2} c_2 + \cdots$.

Using from dependent $H_1, H_2, H_3$ , finally λ is obtained so:

$\lambda = 0.543689 + 0.3859221\, L \frac{1}{\mu} - 4.1928462\, L^2 \frac{1}{\mu^2} + \ldots$

In the following, $\lambda(r_1)$ , $\theta_0$ diagrams is drawn with regard to length of plate for different modes :

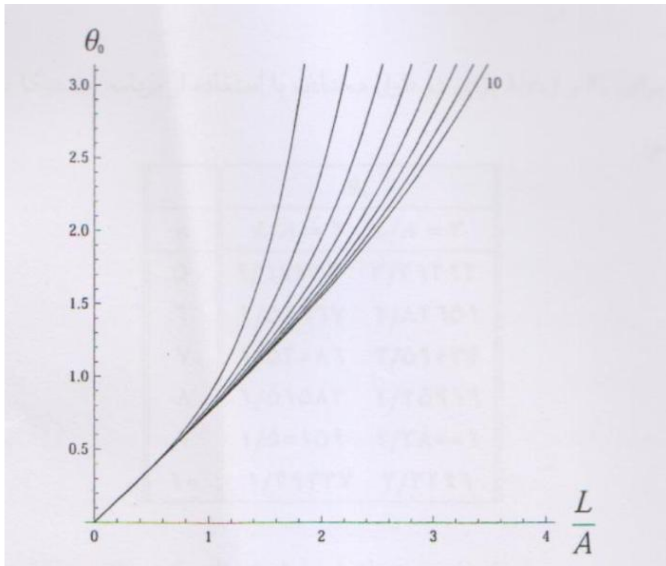
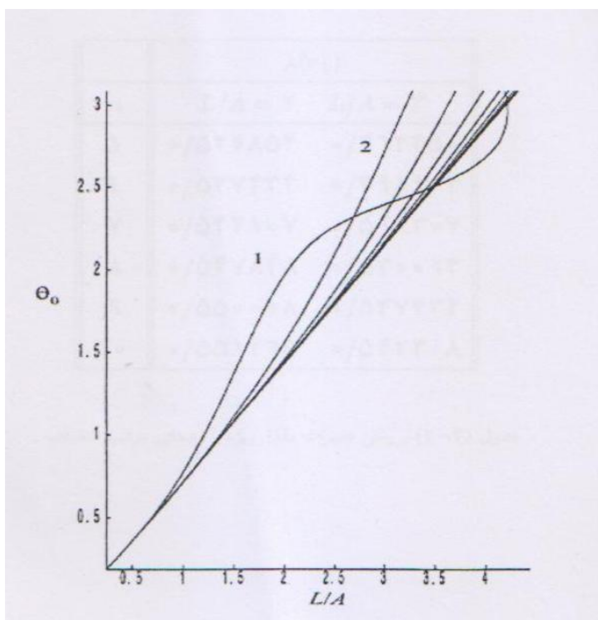



Figure (2). results of WKB method diagram(n=3,4,…,10)




Figure(3). Results of numeral method diagram (n=1,2,3,…,10 )

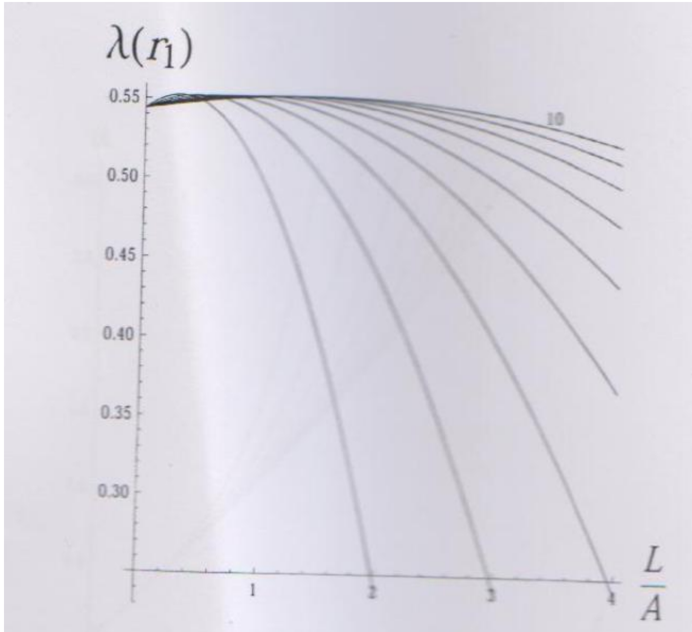


Figure (4) .results of approximate WKB diagram(n=2,3,…,10 )

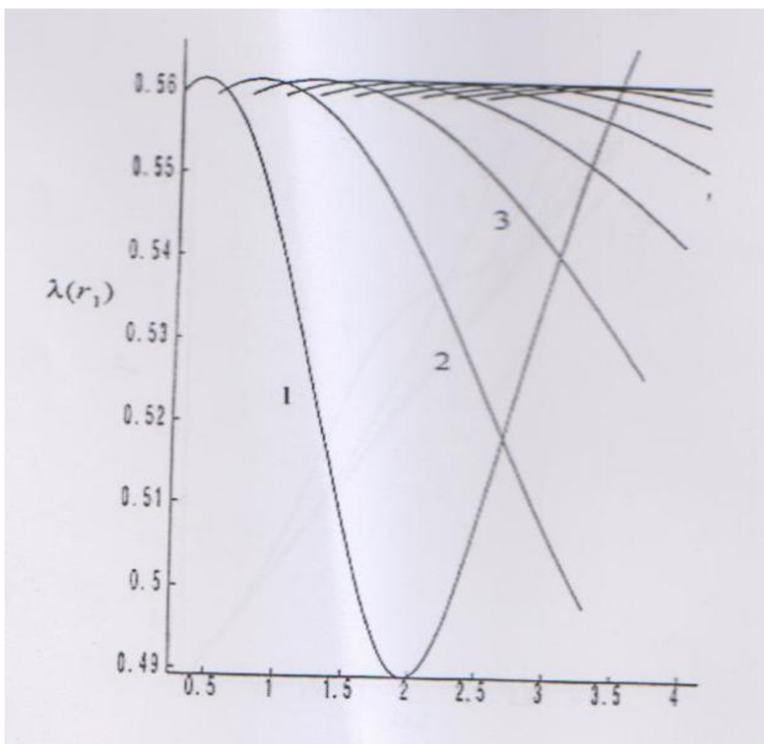


Figure (5). results of numeral method diagram(n=1,2,3,…,10 ) for to different length , we apply $\lambda(r_1)$ ,$\theta_0$ for results of WKB with usage of Mathematics in two table (1) and (2) .

Table 1.An example of critical values of $\lambda(r_1)$ $\theta_0$against undeformed length L/A

| | $\lambda(r_1)$ | |
|---|---|---|
| N | $\frac{L}{A}=2$ | $\frac{L}{A}=3$ |
| 5 | 0.524854 | 0.464458 |
| 6 | 0.537434 | 0.498904 |
| 7 | 0.544107 | 0.518307 |
| 8 | 0.547848 | 0.530014 |
| 9 | 0.550008 | 0.537434 |
| 10 | 0.551265 | 0.542308 |

Table 2.An example of eigenvalues estimates$\theta_0$ against undeformed length L/A

| | $\boldsymbol{\theta_0}$ | |
|---|---|---|
| N | $\frac{L}{A}=2$ | $\frac{L}{A}=3$ |
| 5 | 1.58667 | 3.31492 |
| 6 | 1.58667 | 2.82651 |
| 7 | 1.54086 | 2.59034 |
| 8 | 1.51583 | 2.45916 |
| 9 | 1.50159 | 2.38001 |
| 10 | 1.49337 | 2.3296 |

## RESULTS AND DISCUSSION

As we observed diagrams drawn for two methods with increasing mode become nearer. In figure (4) with increasing (n), (λ) specific amount problem is become homogenous for one specific number (approximate 0.55). In higher modes, our diagrams is approximately direct line and this means that as mode is increased dependence of specific amount to L/A decrease or it doesn't have any dependence. Specific amount Matrix for all diagrams figure (5) is same and we can say that it remains near this point (o.55). But in (2) diagram we are faced with groups of diagrams that are moving to right and down side.

Results shows that for pages with length $0.5 < \mathrm{L/A} < 4$,as mode is increased, bending angle comes lower and curvedness occurs sooner. Numeral methods have application until Numeral methods are useful until vacillations are normal. But when vacillations are high, numeral methods are not useful and so by application of asymptotic methods obtained results are more precise and clear. In this research specific amount problem getting form an elastic page ‘s curvedness which is made of Neo-Hookean ,with using WKB method has solved and results were compared with numeral results [5] , diagrams are conformed on each other . So for solving low vacillations both methods show good and reliable results.

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