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Predictive Control for Non-linear Dynamic Systems Using Volterra Linear Modeling

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ABSTRACT: Non-Linear control is basically important part in engineering control. It has a long background using efficient variety of methods in successful industrial applications. One controller designing method in non-linear control is linearization method which was used successfully in industrial control problems. Predictive control which is a model-based control method is also one of the advanced control methods in practical applications because of its successful reason including capability of implementation in time-domain, capability of employing constraints during designing, and capability of employing to multi-variable systems.This article deals with investigating and implementing predictive control for non-linear dynamic systems using Volterra linear modeling. Volterra model has capability of modeling non-linear systems and can be introduced as an efficient model in predictive control because of its simple use and its capability of modeling non-stable systems. If the criterion function is modified, the optimization of problems will be resolved; such problems as incapability of controlling a stable system which has any zero out of the unit circle and also existing any area which has bad attenuation in the z-plane.
Keywords: non-linear systems, Volterra model, Predictive control, Linearization

INTRODUCTION

Predictive control as one of the advanced control methods is also successful in practical applications insofar as it achieved a lot of popularity in industries and universities in last decades. The most important characteristic and property of this control method which is the main factor in its industrial developmental applications are the functioning principals of its algorithm which is comprehensible and rather simple for engineers and operators. It is an important characteristic of a suitable control method for industrial processes to choose from. The other more important characteristics of predictive control which caused its broader applications are in brief as follows:
designing and implementation in time-domain,
capability of employing constraints during designing,
capability of employing to multi-variable systems, and
capability of employing it to simple systems up to complex systems (M. Jalili Kharajou, 2004,A. Zare, 1994, B. Da Silva1 et al., 2012).Predictive control is belonged to the class of model-based controllers. By using the model of system in this method, output is determined in such a way that the defined criterion function will be optimized and the future system inputs will be specified by solving it. By employing the first control vector component to the system, the whole method will be repeated at the next moment by using the last measured information(A. Zare, 1994, Z.Sinaeefar et al.,2011).As mentioned before, predictive control is a model-based control method; therefore, a model should be considered for the system in order to obtain predicting equations which are key roles in designing the controller. To model the process of control in predictive control, different model such as impulse response model, step response model, and conversion function model were introduced. The following issue in this article deals with introducing Volterra model and how to identify this model. In section three, this issue deals with introducing the modified criterion function, solving optimization problem, and in the end, solving problem’s algorithm of predictive control for non-linear systems based on linearization processing model. In the last section, the issue deals with the implementation of the above process for the typical problem and simulation in order to investigate the accuracy of controller operation.

Volterrra Model

Volterra model was first introduced by a French mathematician named Vito Volterra(J. G. Nemeth et al.,2002). This model was used to analyze non-linear circuits and since then it has been used broadly in small but difficult computing.

A non-linear memoryless system can be defined by Taylor series as:

$$y(t) = \sum_{n=1}^{\infty} a_n [x(t)]^n \quad (1)$$

In the next step, Volterra series can be used to describe a memoryful non-linear system as:

$$y(t) = \sum_{n=1}^{\infty} \frac{1}{n!} \int_{-\infty}^{+\infty} du_1 \ldots \int_{-\infty}^{+\infty} du_n\, g_n(u_1, \ldots, u_n) \prod_{t=1}^{n} x(t - u_t) \quad (2)$$

in which:

x (t) = input system,
y(t) = output system,
$g_n(u_1,\ldots,u_n)$ = Volterra kernels,
u_i = time variables.

Linear discrete Volterra model is generally used in order to model non-linear systems in this article in which output system is modeled as follows:

$$y(k+t) = \sum_{i=1}^{k+t-1} d(i) * u(k+m-i) + \sum_{i=1}^{k+t-1} g(i) * y(k+t-i) \qquad \text{for } t = 1, \ldots, h_p \quad (3)$$

in which :

$$m = \begin{cases} t & \text{if} \quad t \le h_c \\ h_c & \text{if} \quad t > h_c \end{cases}$$

H_c= control horizon,
H_p= prediction horizon.

Controller designing in predictive control has a direct correspondence to the approximation quality of model coefficients. Different methods have been used to identify systems up to now including such as LS (Least Square), Phasor LS, RLS (Recursive Least Square). Using neural network is known as part of these methods. High-order Voletrra series needs many parameters. Even in some cases, a second-order Volterra series needs a cursed dimensional device. Identification method based on input-output data is often suitable which is used in this article.

Solving Non-Liear Predictive Control Problems Using Linearization

As mentioned above, predictive control predicts its future outputs by using system model and specifies control signals by optimizing objective function at any moment(L. Zhijun et al.,2008). If non-linear model is used for non-linear systems, optimizing problem of objective function will convert to optimization of a non-linear function which is time-consuming method; therefore, optimizing problem will convert to linear optimization by finding a local linear model at any moment. Accordingly, non-linear predictive control problem will convert to linear predictive control by using this modeling method. Dynamic conversion of non-linear system in the form of linear is the main idea of this method which is successfully used to solve practical control problems.

MPC (Model Predictive Control) algorithm accompany with constraints of designing has capability of solving optimization problem at any moment by using numerical solving methods dedicated for non-linear model but it is not rapid and valid on the whole. It is natural as mentioned before that process model is one the approaches to overcome this linearization problem. Then the calculation of output signal of controller is achieved by using the linear MPC algorithm.

If we consider linear model for the system and identify the desired order in the form of linear case at the identification stage, the problem will convert to MPC problem with linear model.

As we know, predictive control algorithm based on minimizing an objective function is stable at the prediction horizon. The right choice of this criterion function is considerably important. If the objective function is considered as:

$$J(u(k), \ldots, u\big(k + h_p - 1\big) = \frac{1}{2} \sum_{i=1}^{h_p} [y(k+i) - w(k+i)]^2 \quad (4)$$

in which W is desired output and Y is predicted output. There are some difficulties such as incapability to control the stable system which has zero out of the unit circle, and existing domains which have bad attenuation in the z-plane although being stable. Solving these above difficulties, the modified objective function can be used in such a way that the criterion function includes controller output; therefore, we consider the criterion function as:

$$J(u(k), \ldots, u\big(k + h_p - 1\big) = \frac{1}{2} \sum_{i=1}^{h_p} [y(k+i) - w(k+i)]^2 - \frac{\rho}{2} \sum_{i=1}^{h_c} (\Delta u(k+i)^2 \quad (5)$$

In order to optimize the above criterion function, the following conditions must be satisfied as:

$$\frac{\partial J}{\partial u(k+i)} = 0 \qquad \text{for } i = 0, \ldots, h_c - 1 \quad (6)$$

Therefore we have:

$$\frac{\partial J}{\partial u(k+i)} = \sum_{i=1}^{h_p} [y(k+i) - w(k+i)].\frac{\partial y(k+i)}{\partial u(k+j)} + \rho.\Delta u(k+j) = 0 \qquad \text{for } i = 0, \dots, h_c - 1 \qquad (7)$$

$$\text{error}(i) \triangleq y(k+i) - w(k+i) \quad , \text{jacobian}(i,j) \triangleq \frac{\partial y(k+i)}{\partial u(k+j)} \qquad (8)$$

According to the above definitions, the formula (7) converts to:

$$\frac{\partial J}{\partial u(k+i)} = \sum_{i=1}^{h_p} \text{error}(i).\text{jacobian}()i,j + \rho.\Delta u(k+j) = 0 \qquad (9)$$

Using the offered model for the process in order to calculate Jacobian matrix, we have:

$$\text{jacobian}(1,1) = d(1) \qquad (10)$$

$$\text{jacobian}(m,1) = d(m) + g_1.\text{jacobian}(m-1,1)| + \cdots + g_{m-1}.\text{jacobian}(1,1) \qquad (11)$$

And in order to calculate the other elements of Jacobian matrix, we can use the following formula as:

$$\frac{\partial y(k+i)}{\partial u(k+j)} = \begin{cases} 0 & \text{if } i \le j \quad \text{if the system is causal} \\ \approx \frac{\partial y(k+1)}{\partial u(k+j-1)} & \text{if} \quad i > j \end{cases} \qquad (12)$$

Now, writing output according to unknown parameters of the problem (i.e. control vector), we define the following formula:

$$y(k+i) \triangleq q(i, h_p).u(k + h_p - 1) + \cdots + q(i,1).u(k) + \text{const}(i) \qquad \text{for } i = 1, \dots h_p \qquad (13)$$

in which:

$$\text{const}(1) = \sum_{i=2}^{k} d(i).u(k+1-i) + \sum_{i=1}^{k} g(i).y(k+1-i) \qquad (14)$$

and

$$\text{const}(t) = \sum_{i=t+1}^{k+t-1} d(i).u(k+t-i) + \sum_{i=t}^{k+t-1} g(i).y(k+t-i) + \sum_{i=1}^{t-1} g(i).\text{const}(t-i) \text{ for } t = 2, \dots, h_p \qquad (15)$$

and also

$$q(t,n) = \sum_{i=1}^{t-n} g(i).q(t-i,n) + d(t-n+1) \qquad \text{for} \quad n = 1, \dots, h_p \quad , t = n, \dots, h_p \qquad (16)$$

To sum up the above formulas, we conclude the following equation as:

$$\overrightarrow{A}.\vec{U} = \vec{P} \qquad (17)$$

in which P and A matrices are fixed matrices having $A_{h_c * h_c}, P_{h_c * 1}$ dimensions consequently; therefore, the unknown control vector can be obtained as:

$$\vec{U} = \vec{A}^{-1}.\vec{P} \qquad (18)$$

According to this description, solving problem algorithm can be formed as follows which you:
Identify it as an offered linear model according to the inputs and outputs the system had (defining d(i) and g(i) coefficients)
Obtain optimized control in order to achieve the desired output on the predicted horizon and control horizon by solving MPC problem
Apply the first obtained control component to the system and calculate the outputs.
Repeat the previous steps consequently for the wanted time according to time advancement on a scale of one unit ahead.

SIMULATION OF RESULTS

To validate the control algorithm, we show a controller's simulation on non-linear system with the following parameters as follows:

$$y(k+1) = u(k-1) - \frac{a.y(k-1).u(k-1)}{1+y(k-1)^2} + 0.3\frac{y(k-3)^2}{1+y(k-2)^2} + \sin\left(\pi * \frac{k}{50}\right).u(k-d)^2 \qquad (19)$$

in which:

$$d = \begin{cases} 0 & \text{if } k < 40 \\ 2 & \text{if } 40 \le k \le 90 \\ 0 & \text{if } k > 90 \end{cases} \quad a = \begin{cases} 0.25 & \text{if } k < 100 \\ 0.15 & \text{if } 100 \le k \le 200 \\ 0.5 & \text{if } k > 200 \end{cases} \quad b = \begin{cases} 0.5 & \text{if } k \le 70 \\ 0.65 & \text{if } 70 < k \le 160 \\ 0.75 & \text{if } k > 160 \end{cases}$$

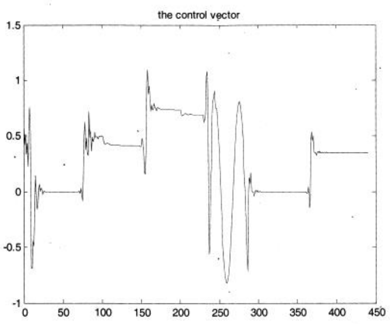

Figure 1. controller output

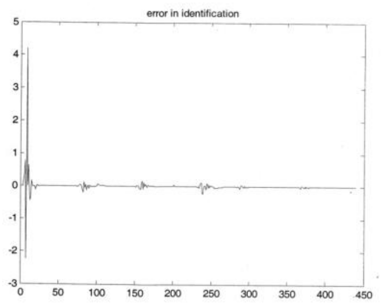

Figure 2. system error in identification

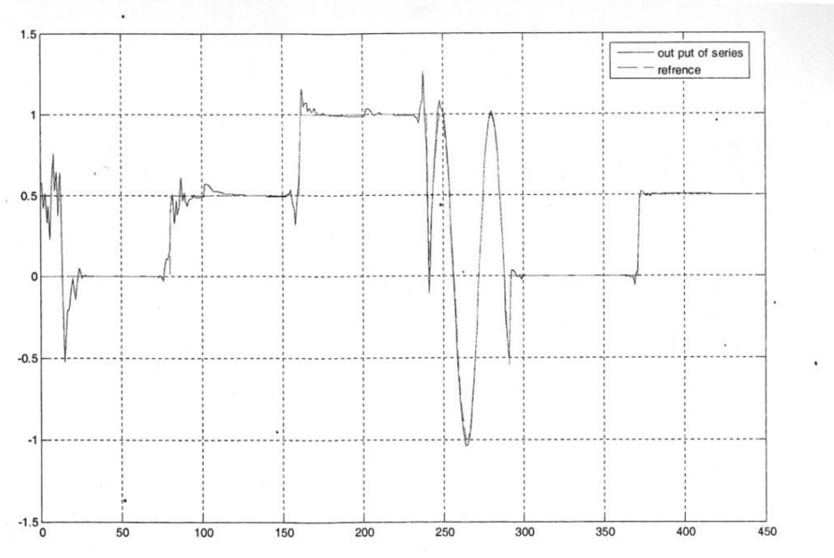

Figure 3. system output and base route

The stated algorithm caused a significant decrease amount of calculation in optimization case and has capability of modeling unstable system. A variety of tracing of a vast spectrum of signals is possible by using this method. It has also capability to overcome the structural changes in the model, especially in delay changes. In addition to the changes of parameters of (a, b) models, in case of an important parameter change such as (d) delay and according to the change of some stated parameters, the simulation shows that the response resulted from the predictive control algorithm can tolerate these changes well and trace the desired route well.

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