Reviews the Performance of Robust Portfolio Optimization using Maximum Sharpe Ratio compared to Classical Optimization Model

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ABSTRACT: In fact optimization portfolio and diversity are the bases to develop and improve the financial markets and also the financial decisions. In the present research the function of the classic optimization portfolio (Markowitz Mean-Variance Model) has been studied as the targeted function in the presence of the Sharp Ratio. This study has presented a stable developing model of portfolio in which the results of the optimization portfolio have been shown by the relevant corresponding stable formulizing procedure based on the Factor Model by using the stock market indexes as the input data and the test of the stability of the input parameters. To do this the input data were chosen in a given confidence interval based on the worst predetermined scenario in order to maximize the Sharp Ratio. In the present survey there has been an attempt to have a survey on 164 monthly portfolio of all the accepted firms in Tehran Stock Exchange Market during a span of 14 years and the risk and return potentiality of each portfolio has been calculated based on both models of the Classic and Optimization Sharp Models. In the next level, a detailed survey has been conducted to investigate the probability of the existence of a significant difference between the risk and the real return potentiality of these two models by using the mean-difference test. The results indicates that the real return and real risk in the Sharp Model Respectively has no significant differences and has significant differences in the classic model.

Key words: Risk, Return, Mon-factor Model, Stable Optimization, Sharp Ratio

INTRODUCTION

Problem of choosing portfolio, to consist of: circumstance to allocate capital to numbers of available assets for sake of taking to output maximum with reach to minimum risk in synchronous time. Although, to be comforted attention to making various expenses in risk deduction in start of financial markets, but first of mathematical model portfolio election was instrumental formula in 1952 by Markuites. In Markuites portfolio choice model, (efficiency) of a portfolio, to measuring same as collision changeable expectation value (portfolio efficiency) and (risk) portfolio is calculation by portfolio efficiency various. Markuites showed, in supposition, if capitalist accept a high level of risk (in other words capitalist is risk full) or capitalist wants lower littoral of efficiency (capitalist being to run away of risk), optimized portfolio can be result of second rank question solution. This average - various model have effect so profound in financial markets economic model and expansionist capital assets price model (CAPM) by sharp.

Literature Review and Record Research

With publication Markuits article, by (portfolio choice) label in 1952 he can to pace first and great in insufficiency portfolio management. Markuits opinion to be comforted base of portfolio selection, today this object is to point by plural of the word same as various-average analysis, various-average optimized, and portfolio modern model. Then of outstanding attention points in Markuits model, are, attention to risk in investing not only are based on risk bonds one by one, suppose they are based on risk of investment collection. Markuitses model was based on under data:
All of capitalist have expectation addition desirable, and their desirable limit curve is deduction dependant and all of capitalists are trying to making maximum desirable.

All of capitalist are logical and to run away of risk.

All of capitalist have access of the required size of information and in the contemporaneous.

All of assets have subscriber of normal distribution.

All of assets have steady and certain relation with each other.

All of capitalist choice themselves portfolio based on expectation average and various. Therefore, their irregular curves are dependent of efficiency rate and expectation various.

All of capitalist have horizon time for one period.

Before publication this paper by Hery Markuites in 1952, all of capitalist had acquaintance with making miscellaneous concept. The fact that,' do not set your eggs in one basket' is primitive proverb, who to make a sign to make miscellaneous. But Markito for first time polished to scantiness make miscellaneous concept by statistical purports.

Central limit theorem is basic bullion in making various concept. The receptacle Central limit theorem, totally of dependent changeable and independent with limited various, are one Gaussian distribution. In simple shape, we can show this proposition like bottom. According this proposition, if being chancy independent changeable and each have a distribution optional probability with average and various ; Then:

\[
\lim_{n \to \infty} P \left( \frac{1}{\sigma \sqrt{N}} \sum_{i=1}^{N} (X_i - \mu) \leq y \right) = \frac{1}{\sigma \sqrt{2 \pi}} \int_{-\infty}^{y} e^{-\frac{1}{2} s^2} ds
\]

With to Have top cases, for each portfolios to have asset with efficiencies, in the event that measure of each one of assets investing being equally, portfolio efficiency is similar under:

\[
R_p = \frac{1}{N} \sum_{i=1}^{N} R_i
\]

On the basis of Central limit theorem, portfolio’s various will similar under:

\[
\text{var} (R_p) = \frac{1}{N^2} \sum_{i=1}^{N} \text{var} (R_i)
\]

\[
= \frac{1}{N^2} * N * \sigma^2
\]

\[
= \frac{\sigma^2}{N} \text{~} N \to \infty \to 0
\]

In the event that number of assets, of the required great size, portfolio’s efficiency of collision changeable will with Gaussian distribution. Conformity of top relation, with increase assets number, portfolio’s various being decrease and in result being zero. Of course, this situation is completely ordinary, because in this actual world for the reason that, there are to collude between assets, we can not with increase in assets amount, have expectation of a portfolio with zero various.

**High Modern Portfolio Model**

First time Rome and Ferguson (1993 ) in one article with label of ‘ portfolio up and down modern model being evaluation ’, up and down modern model terminology have been application in literature on the outside. In this paper, the investigators told ‘ modern portfolio recommendation have not satisfactory in special conditions and they to believed and knew unfit operation were arising of two assumption in this model:’

1. Various of efficiency is correct criterion for measuring risk investment.

2. Distribution of efficiency totalizes of assets and valuable bonds are normal.

In first assumption, with calculation various as risk criterion, by all species constructive and negative eventual vacillation in future, are risk. Original declaration in this criterion is concurrent themselves regard to great efficiency of expectation efficiency and small efficiency. In fact of vision of investor to take great efficiency fortunate

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expectation efficiency not only are not undesired, suppose investor being going to meet, especially in bull market, investor are following researching accept of top and great efficiency. 

Most of concepts and models production were in length 60 years ago, in assumption being normal in efficiency distribution in assets and value leaves totalizes is solid. But except some exception, totally of financial considerations to be fulfilled not have been support of 1960 until now.

Winter Famma (1963), and Mandelbrot (1963), were first investigators in separate researches, did in case of efficiencies distributions stokes, they rejection normal assumption efficiencies distributions stokes. Most of done experimental researches are show a efficiency time series have fat tails and skew Rachev S, Menn C, Fabozzi F,(2005).

Fat tails is leptokurtosis, which efficiency distribution of finance assets is louder than normal distribution and idiomatically they have fat tails. This means large efficiency (constructive and negative) which idiomatically happened to calls Extreme event. Statistical intelligible is same as normal distribution which to base of fat tails and they can estimate good data distribution near to central limit theorem and they can not using of new observations in fat tails. Of course which this job involved problem, of knew statistical distribution same as normal distribution to ascertain trail act respect.

Also in additional to this two assumption which were demanded by Rom and Ferguson, most of another supposed modern portfolio model and another that result models same as capital assets price model, are not true and were been demanded too. 

Resembling to include certainties which were conclusion in recent years on the optimized stable in portfolios management, is explanation under: Hodges (1998), Zakamouline and Koekbakker (2009), they were to define the total stable sharp respect (general). This crooked respect (third rare) and protraction (fourth rare) are enter observations in the historic efficiency distribution.

Lo (2002), statistical behavior sharp respect is under this observation which we can take result of normal efficiency distribution. Mertens (2002), he showed and deployment weak Lo result with normal efficiency distribution assumption. 

Problem of portfolio choice with using VARSR to produce as a sample of optimized portfolio. Stable portfolio optimized takes this confidence, efficiency distribution to estimating under study in each moment by historical efficiency.

Goldfarb (2003), Iyengar (2003), they to defined unconfident structure concept (or non-final nature structure) for various and expectation efficiency estimate and they showed fact of efficiency of stable portfolio allocation in a certain confidence level. Making maximum is more pessimistic than sharp respect, one of stable portfolio optimized which were production by Goldfarb and Ingar in 2003.

Koeni and Tutuncu (2004), they to extended this method and they supported of this plan ‘choosing conservativeness portfolio, have effect when the resulting to increase in portfolio efficiency in more pessimistic of scenario’. These two writer were been using as a general rule of non-confidence sets of imported parameter for efficiency distribution in making model and also, they were been using of distinct non-confidence sets for distribution average and various estimated.

Christie, (2005), Opdyke (2007), they changed normal (dependent) and independent efficiency assumption (I.I.D) to stable and Orgudic(6) efficiency. These two men showed, the estimate regard sharps are correlated series even they have efficiencies and or conditional vacillation have normal distribution with time. These results are organizing VARSR so straightly.

5. Assets efficiency are independent under the study and exactly they have normal of distribution collision changeable.
6. Orgudigh recommendation is connected to tributary of mathematics streamer, which this vivid systems is to consider with a fixed criterion and affairs related to them.
DeMiguel; Nogales (2009) , they used of stable estimating, M estimating , S estimating except pessimistic scenario and they showed external sample particulars. For more acquaintance in this basis recent searches, we can looking to activities research in choosing stable portfolio in 2010 by Fabozzi.

Zymler , ( 2011 ) , he recent added a portfolio insurance guarantee to optimized portfolio with differential coefficient sheets in portfolio stable optimized governor-general form as covering immoral accident addition market.

Bertsimas , ( 2011 ) , he did production a general passage to making optimized literature, when Ben-Tal ; Nemirovski , (2007 ) , they worked special to stable crooked making optimized of discourse in a summary look.


KhaluZadeh , and Amiri , ( 2007 ) , they did production to the model for making optimized portfolio with using of Genetic Programming. In this model, value focused in as estimate risk criterion at stake risk . The investigators did application the produced model for framing portfolio organized of 12 share of different companies in Tehran stock exchange. Results were in stable of risk, when market risk making model of efficiency method to basic value recommendation are in raged and Genetic Algorithm optimized method in taking optimized weights of shares basket with in looking to limitation.

Abbasi and et al, (2010) , they did calculation value in risk reagent with using of the parametric, for 100 active company in Tehran stock exchange and they did addition as a limitation to Markuits sheets model and they organized different optimized portfolio with alteration in value parameters in accepting risk reagent in investment and accepting its confidence per cent . This paper result showed, adding value limitation in risk reagent to Markuits model, perhaps limited useful frontier, changing to a spot and or decadence them.

HanafiZadeh and et al, (2007), they did their time to using of separation eventual space method to presentation a model for choosing the optimized portfolio with looking criterion value in risk reagent, offered method by these researcher were not in base of scenario and this model using directly of making maximum total investment efficiency rate on the basis of value in risk reagent. Also researcher offered another model for capital allocation with using of dramatist MontKarlo method. Researcher performance of this offered model was this model can not grown up contrary to another basis on scenario models.

Ghadiri Moghdam and Raffieai Darani, (2011), they did appointment and studied optimized portfolio on the subject of active companies sheets in nutritious industries in Tehran stock exchange with using of value in risk reagent. Researcher used of sheets weekly statics of activity companies of nutritious industries in Tehran stock exchange of 2009 September until 2011 February for done this research. In this paper, they used of correct planning method for making optimized. Continuously there are tolerable result in this research as directly connection between value in risk reagent and capitalists expectation efficiency, although there are not direct relation between value in risk reagent and variety portfolio.

Farid and et al, (2011), they did presentation a model for choosing optimized portfolio with using of value in risk reagent and exploitation of MontKarlo dramatist model. In this research, first they did calculation value in risk reagent’s several sheets with using of MontKarlo dramatist model and end of all, they appointed investing optimized synthetic model application in dread.


**Community, Sample and Statistical Method’s Research**

For the purpose of production of investment portfolio choosing based on optimization with using of stable sharp relation must using of company data. To this arrangement, statistic community research of information are base on all companies in Tehran stock exchange and for the purpose of , evaluation sample research model, this research data are studying for 15years of 2010 February until 2016 February. In this paper, required collecting information, hypothesis and theoretical argument research, using of library method and based on all of company data in Tehran stock exchange and specially they have suitable reliability and validity. In this manner, in this paper, for measurement portfolio optimization efficiency using with stable sharp relation in comparison with classical optimization portfolio we producing Tehran stock exchange information in one sample and we recognize optimization in two form.

**RESEARCH METHOD**

**Classical Model Choosing Portfolio**

In this frame, we imaging a capitalist who want to organize a portfolio with N risky asset. $w = (w_1, w_2, ..., w_N)/$.This formula is showing capitalist weight bearer in each of assets. We are assumption
assets efficiency bearer is same as this bearer → $R = (R_1, R_2, ..., R_N)$, in this situation bearer of expectation assets efficiency is as → $\mu = (\mu_1, \mu_2, ..., \mu_N)$. $\Sigma$ is various – co various matrix of assets efficiency, which were defined in under:

$$\sum = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \vdots & \sigma_{21} & \sigma_{22} \\ \vdots & \vdots \\ \sigma_{N1} & \sigma_{N2} \end{bmatrix}$$

Relation 4

Which in that:

$\sigma_{ij}$ is showing, co varicose are between I and J, which is on the basis of front relation $\sigma_{ij} = \sigma_i . \sigma_j . \varphi_{ij}$, $\varphi_{ij}$ is correlated coefficient between two assets efficiency of I and J, and Also $\sigma_{ii}$ is $\sigma_i$ square root. Mean: $\sigma_{ii} = \sigma_i^2$

With due attention to top supposed, question of making classical average-variables optimized was produced by Markuits as under:

$$\mu_p = \frac{W}{\mu}$$

Relation 5

$$\sigma_p^2 = \frac{W}{\Sigma W}$$

Relation 6

For example, if capitalist investing only in two assets with this weights → $W = (W_1, W_2)$, expectation efficiency rate and portfolio’s various is calculation in under:

$$\mu_p = W_1 \mu_1 + W_2 \mu_2$$

Relation 7

$$\sigma_p^2 = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$$

$$= [W_1 \sigma_{11} + W_2 \sigma_{12}, W_1 \sigma_{12} + W_2 \sigma_{22}]$$

$$= W_1^2 \sigma_{11} + W_2^2 \sigma_{22} + 2 W_1 W_2 \sigma_{12}$$

Relation 8

Therefore portfolio’s optimized is calculation in classical Markuits model outwardly under:

$$\min_W \frac{W}{\Sigma W}$$

s.t: $\mu_0 = \frac{W}{\mu}$

$$W/\tau = 1$$

Relation 9

Relation 10

Relation 11

This begging is a quadratic optimization begging.

$$W = g + h \mu_0$$

Relation 12

In this formula $\mu_0$ target is expectation efficiency rate and there are under relations:

$$g = \frac{1}{ac - b^2} \Sigma^{-1} [c\tau - b\mu]$$

Relation 13

$$h = \frac{1}{ac - b^2} \Sigma^{-1} [a\mu - b\tau]$$

Relation 14

$$a = \tau/\Sigma^{-1} \tau$$

Relation 15

$$b = \tau/\Sigma^{-1} \mu$$

Relation 16

$$c = \mu/\Sigma^{-1} \mu$$

Relation 17
\[ N \sigma^2 = \frac{\sigma^2}{N_{N \to \infty}} \to 0 \]  

Relation 18

**Sharp Model Choosing Portfolio**

One of models, that is near to classical average-variables model is choosing optimized portfolio via of making relation optimized sharp. Relation sharp is criterion for weighing portfolio efficiency, which was innovation by sharp in 1966, and in the that basis is line of capital market. For calculation this relation we division using risk portfolio to standard digression. In fact with using of relation sharp we are trail to calculator consumed amount that a capitalist receipt front of tolerance full risk. Sharp is calculation with under formula:

\[
S_{RP} = \frac{\mu_p - \bar{\tau}_f}{\sigma_p}
\]

Relation 19

In this formula, \( \bar{\tau}_f \) is efficiency rate of with out risk.

Each measure of sharp relation was calculation for a portfolio is more, that portfolio have better efficiency, for this reason if we make maximum in the sharp we arrive to one sharp which have suitable efficiency and we can have hope that in future we will have suitable efficiency. Classical making relation sharp optimized problem is as under:

\[
\max_{\mu \in \mathbb{R}^N} \frac{W \mu - \bar{\tau}_f}{\sqrt{W \sum W}}
\]

Subject to:

\[ W_i = 1 \]

Relation 20

\[ w \psi_{i,j} \geq -W_i, i = 1, ..., N \]

Relation 21

If we assumption this position of making stable optimized of average-variables is a structure for expectation efficiency and co various and if we did application the non-definite sets which effect in stable optimized average-variables to defined for expectation efficiency and co various matrix, then making stable optimized shop relation will be same as question of making stable average-variables in under form:

\[
\max_{w \psi_{i,j}, \tau, \eta, t_j, v} \frac{2 \sigma^2}{\sqrt{\sum W}}
\]

Subject to:

\[ s.t. W_i = 1 \]

Relation 22

\[ \psi_{i,j} \geq -W_i, i = 1, ..., N \]

Relation 23

\[ \tau + t_j \leq v - k \]

Relation 24

\[ \eta \leq \frac{1}{l_{\max}} (H) \]

Relation 25

\[ \left[ \begin{array}{c} 2 \psi^2 \psi \psi_{i,j} \\ \eta - \tau \\ 1 - nl_i - t_j \end{array} \right] \leq \eta + \tau \]

Relation 26

\[ \left[ \begin{array}{c} 2 \psi^2 \\ 1 - nl_i - t_j \end{array} \right] \leq \eta + \tau \]

Relation 27

Which in that:

\( M \) is number of skill model.

\( Q \) is this \( H = G^{-1/2} FG^{-1/2} \) spectral analysis,

\( L \) is a diameter matrix with \( l_1, ..., l_m \) diameter range (\( l_{\max} \) is maximum of diameter range) and

Also \( S \) is equal with \( s = \hat{Q} H^{1/2} G^{1/2} V W \).

Question of top optimization, as average-variables optimization which in first time is very intricate, is a quadratic con question (SOCP) that simplicity we can being optimize this position with using of dissolving software. But main question in this research is ‘maximum question of stable relation sharp’. In this paper choosing portfolio is our goal, which is pessimistic scenario is when expectation efficiency making maximum in a extra portfolio regard to with out risk efficiency, this mean is making maximum relation to standard digression. Equivalent making maximum–minimum have related to under formula:

\[
\max_{\phi: \tau^T \phi = 1} \min_{\mu \in \mathbb{R}^N, \nu \in \mathbb{R}^D, \delta \in \mathbb{R}^D} \left\{ \frac{E \tau_0 - \bar{\tau}_f}{\sqrt{\text{var} \tau_0}} \right\}
\]

Relation 30
We assume value optimization is positive in making maximum – minimum, in other words there is a portfolio with limited pessimistic various which that efficiency in worth position of rate with out risk efficiency of $r_f$. In operation pessimistic various of each asset have littoral, therefore this condition situation being decrease to this necessity which minimum there is an asset which that efficiency are more than $r_f$ in pessimistic situation. Maximum stable relation sharp mean stability equation was to explain in under:

$$\max_{\{\phi \geq b \}, 1^T \phi = 1} \min_{\{ \mu \in s_m, V \in S_v, \rho \in S_d \}} \left\{ \frac{\mu^T \phi - r_f}{\sqrt{\text{var} [ \phi ]}} \right\}$$  \text{Relation 31}

This formula is equal with minimum of stable various (of course with more stipulation):

$$\text{minimize} \quad \max_{\{ v \in s_v \}} \phi^T V^T F V \phi + \phi^T D \phi$$ \text{Relation 32}

$$\text{subject to} \quad (\mu_0 - \gamma - r_f^1) \geq 1$$ \text{Relation 33}

$$A \phi \geq \xi b$$ \text{Relation 34}

$$, \quad 1^T \phi = \xi$$ \text{Relation 35}

$$\xi \geq 0$$ \text{Relation 36}

Which $\xi$ is an auxiliary changeable, this changeable being introduce for $1^T \phi = 1$ and $A \phi \geq b$ fellowmen.

Now top minimum stable various problem changing to a second degree conic planning problem. As under form:

$$\min_{\mu, \tau, \ell, v, \delta, w} v + \delta$$ \text{Relation 37}

$$\text{subject to} \quad \| D^{1/2} w; 1 - \delta \| \leq 1 + \delta$$ \text{Relation 38}

$$\| M w \| \leq (\mu_0 - r_f)^T w - 1$$ \text{Relation 39}

$$\left( p^T ; v ; w \right) \in H \left( V_0, F, G \right)$$ \text{Relation 40}

Primary and Secondary Hypotheses

Hypotheses of research is based on research goals and rejected of research question. Just as, in this research, the goal is comparison two criterion efficiency and portfolio risk in making classical portfolio optimized and stable sharp, therefore research hypotheses are in under:

Primary Hypotheses

Choosing optimized portfolio model is efficient on the basis of sharp relation in making stable optimized mould relation to Choosing optimized portfolio model.

Secondary Hypotheses

Portfolio real efficiency have sense difference with foresighted portfolio efficiency on the basis of making classical optimized basis on making stable sharp optimized.

Portfolio real risk have sense difference with foresighted portfolio risk on the basis of making classical optimized basis on making stable sharp optimized.

In as much as research target is choosing risky portfolio optimized based on stable classical and sharp, in basis time period accepted company in Tehran stock exchange were in good shape of cashable arrangement. Afterwards each company have minimum condition of suitable cashable, are choosing caption of company present in portfolio in one month span. This is important, one of the enter changeable is error stable level sharp, that this position taking of 1% level. Therefore each of these portfolios were comforted to test in stable sharp method in a level.

CONCLUSIONS RESEARCH

Choosing portfolio is one of the question which to break out mind of most capital market active person. That which being distinct different optimization portfolio model is criterion of risk calculation. Risk of investing is one of the important question which capitalist confronting on them. As a general rule, capitalist are trail acquisition to most efficiency with minimum risk. Therefore one of the important challenge to portfolio framing, is fixing relation or optimization weight of existing shares, in portfolio for making risk minimum. Skill models and specially skill single model forming on basic of new definition of risk, this research is using of risk foresight and efficiency in form of skill.
single model for portfolio optimization. Conformity of definitions, intellectual capitalist being investing in one of these two figure:
In investment position, investor choosing situation similar risk with more efficiency.
In investment position, investor choosing situation similar efficiency with less risk.
Result of this paper is agreeable with second case. In other words, although this paper were show that choice portfolio efficiency was similar on the basis of maximum model of stable sharp with efficiency of classical model, but have less risk and can good way in investment activity.
At the end suggested under case to researcher:

We can spend to comparison foresight efficiency with actual efficiency and foresight risk with actual risk in optimization stable sharp model.
Using of optimization stable sharp risk and portfolio efficiency with another portfolios model same as Genetic Algorithm, disturbance theoretical, and nervous network.
We can work on to formation stable sharp portfolio of another criterion risk same as value in risk reagent and non-desirable risk.

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