Design of NARMA L-2 Control of Nonlinear Inverted Pendulum

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ABSTRACT: An inverted pendulum is a pendulum which has its center of mass above its pivot point. It is usually applied with the pivot point mounted on a cart that can move horizontally and may be called a cart and pole. The inverted pendulum problem is one of the classic and commonly used problem in the control engineering field. In this paper a nonlinear technique based on NARMA L2 neuro-controller is applied to control a nonlinear inverted pendulum. Inverted pendulum is oscillated from its unaffected position and stabilized at the desired position. Then, it is used as a controller. The proposed method is successfully applied to control the inverted pendulum. Matlab is used to simulate and analyse the performance of the control schemes. The simulation results shows the advantage of the proposed NARMA L2 method.

Keywords: Inverted pendulum, Nonlinear system, Narma L2 controller, Neural network

INTRODUCTION

The single inverted pendulum is a classical problem in the field of control engineering (H. Kwakernaak and R. Sivan, 1972). Generally, an inverted pendulum consists of a cart and a pendulum jointed to it as shown in figure 1. By implementing a force to the cart, for example through a built-in electrical motor, and thus moving it backwards and forwards will make the pendulum to swing.

Although, dynamic modeling of inverted pendulum yields a highly non-linear problem. The main purpose of the controller in here is to perform the right amount of force which will stabilize the pendulum in the upright position. This process can either be applied with the pendulum being held upright or through an upswing manouevre.

One technique to find a proper control law is to linearize the system equations around the equilibrium point. But the linearized model does not characterize the real system for larger deviations this is not a proper technique for a complete upswing and stabilization control. However, an independent second controller can be designed which performs the upswing movement and then hands over to the linear controller.

A direct way for swinging the pendulum to the desired position is to consider the energy stored in the system and comparing it with the value which corresponds to the maximum height (Karl Johan, and Furuta. 2000).

This energy control technique yields a switching control law, similar to the design of a sliding mode control (Banrejee and M. J. Nigam, 2011).

Generally, to overcome the linearity issues, the control technique is designed as it does not only eliminate the nonlinearity, but also the dynamics of the system.

The closed loop system becomes an algebraic relation between the output and control effort. The position reference can be utilized as reference directly. In such a way, we need only the position reference and the system still perfectly tracks a smooth trajectory. This technique extends higher complexity into the state feedback controller but simplify everything else. Here, the controller is designed based on discrete-time neural network model in order to decrease an effort to model the nonlinearity and to form the controller (Srakaew et al., 2010).

Discrete time neural network is trained offline by using the data pairs from the operation of the system and afterwards formed as controller.

The concept of NARMA L2 controller is based on the illustrated technique and is a proper candidate for controlling a nonlinear system.
In this paper, the NARMA L2 is designed to control the nonlinear pendulum plant. NARMA L2 is based on neural network and is trained based on the input-output data pairs, the nonlinearity and the dynamics of the system is simple and efficient modeled in a single activity.

Modeling of Inverted Pendulum

For modeling the inverted pendulum system, a horizontal movement electrical car composed by the single inverted pendulum can be considered (Komine et al., 2010; Diao and Ma, 2006; Driver and Thorpe, 2004).

\[ \begin{align*}
N & = m_p \frac{d^2}{dt^2} (p + l \sin(\theta)) \\
& = m_p \frac{d^2}{dt^2} (\theta \cos(\theta) - l \sin(\theta)) \\
P - m_p g & = m_p \frac{d^2}{dt^2} (l \cos(\theta)) \\
& = m_p g + m_p l (-\sin(\theta) \theta^2 \cos(\theta))
\end{align*} \]

The center of mass for pendulum is balanced in the moments as below:

\[ I \theta'' = P \sin(\theta) - Nl \cos(\theta) \]

By substituting Eq. (2) into Eq. (1):

\[ F = (m_c + m_p) \theta'' - m_p l \cos(\theta) - m_p l \sin(\theta)) \theta^2 \]

Substituting Eq. (2) and (3) into Eq.4 results:

\[ \theta(1 + m_p l^2) = m_p g \sin(\theta) - m_p l \theta \cos(\theta) \]

For simplifying eq. (5), we have:
\[ M = m_c + m_p \]
\[ L = \frac{I + m_p l^2}{m_p l} \]

(6)

Relationship between voltage and force in the motor cart can be illustrated as below:

\[ F = \frac{K_M K_y}{Rr} V - \frac{K_m^2 K_y^2}{Rr^2} \phi \]

(7)

By eliminating \( \phi \) from Eq.(5), \( \phi \) from Eq. (6), and by replacing M, L and the equation for the force results:

\[ \phi = \frac{m_p l \cos^2(\theta)}{L} = K_M K_y \frac{V}{Rr} - \frac{K_m^2 K_y^2}{Rr^2} \phi \]

(8)

\[ \frac{-m_p l \cos(\theta) \sin(\theta)}{L} + m_p \sin(\theta)(\phi^2) \]

\[ \frac{-m_p l \sin^2(\theta)}{M} = g \sin(\theta) - \frac{m_p l (\phi^2)}{M} \cos(\theta) \sin(\theta) \]

(9)

\[ \frac{-\cos(\theta)}{M} \left( \frac{K_M K_y}{Rr} V - \frac{K_m^2 K_y^2}{Rr^2} \phi \right) \]

\[ \text{Narma L2 Neurocontroller} \]

NARMA is a discrete-time which illustrates the nonlinear dynamical system in neighborhood of the equilibrium state. Generally, an identical NN model of the system which needs to be controlled has to be realized. Subsequently, a developed NN model can be then used to train the controller. NARMA L-2 is a technique to simply a readjustment for the plant model in an off-line training mode with batch process. NARMA L-2 needs the least computation rather than the other neural architectures because the only computation is forward pass through the neural network controller. For an \( n \) order nonlinear SISO system with a relative degree \( d \), the companion form of NARMA can be describes as (Middleton and Goodwin, 1988):

\[ y(k+d) = F[y(k), y(k-1),..., y(k-n+1), u(k),..., u(k-m+1)] \]  

(10)

Here \( u(k) \in \mathbb{R} \) is the control effort sequence, \( y(k) \in \mathbb{R} \) is the output sequence and \( F : R^{2n} \rightarrow R \) and \( F \in C^\infty \).

To train NN for identification part, the nonlinear function \( N \) is approximated. When the system output followed the reference trajectory \( y(k+d) = y_r \), (k+d), nonlinear controller can be developed by using the equation below:

\[ u(k) = G[y(y(k), y(k-1),..., y(k-n+1)), u(k),..., u(k-m+1)] \]  

(11)

Because of some reasons like: dynamic back propagation training and creation of a function ‘G’ for minimizing the mean square error Implementation, the NARMA L-2 Controller is little slow. Since, the solution can be approximated as below:

\[ y(k+d) = f[y(y(k), y(k-1),..., y(k-n+1)), u(k),..., u(k-m+1)] + g[y(y(k), y(k-1),..., y(k-n+1)), u(k),..., u(k-m+1)] \]  

(12)

The achieved model is now in companion form (Hagan et al., 2002), where the next controller input \( u(k) \) is not confined inside the nonlinearity. This form of control input makes the output to follow the reference, i.e. \( y(k+d) = y_r \). The obtained controller can be illustrated as:

\[ u(k) = \frac{y_r(k+d) - f[y(y(k), y(k-1),..., y(k-n+1)), u(k),..., u(k-n+1)]}{g[y(y(k), y(k-1),..., y(k-n+1)), u(k),..., u(k-n+1)]} \]  

(13)

By utilizing the illustrated equation, we can achieve the realization of problems, as the control input \( u(k) \) has to be nominated in order to the output \( y(k) \), at the same time. Therefore, the developed model can be used instead of the previous model as below:

\[ y(k+d) = f[y(y(k), y(k-1),..., y(k-n+1)), u(k),..., u(k-n+1)] + g[y(y(k), y(k-1),..., y(k-n+1)), u(k),..., u(k-m+1)], u(k+1) \]  

Here, \( d \geq 2 \). Figure 2 shows the structure of the NN representation for NARMA L-2.
Nowadays neural networks are introduced as a widely used technique for identification of nonlinear systems. NARMA L-2 has essentially two main steps: The first step is to identify the system which includes developing an approximate NN model of the NLDS. The second step is to achieve NN model of the system to train the controller. The model of a NARMA L-2 for controlling the considered nonlinear system is shown in Figure 3.

An adequate number as training samples (10000) were generated with the above specification. The number of epochs for the mean squared error (MSE) for minimizing was set to 1000; the training algorithm used for the nonlinear inverted pendulum was Levenberg-Marquardt (trainlm) technique.

**SIMULATION RESULTS**

In this paper, for model simulation, analysis, and control of nonlinear inverted pendulumcart dynamical system, Matlab/Simulink models are developed. Here, we provide some experimental results from implementing the Narma L2 controller illustrated above to the inverted pendulum. Figure 4 shows the initial position, angle and input signal of the cart is in the centre of the beam.
Figure 4. Initial state of angle $\theta$, cart position $x$, and control force $u$ for the inverted pendulum system.

The object is to achieve the following constraints: the input is constrained via $|u(t)| \leq 0.5$ and the cart position is constrained via $|p(t)| \leq 0.7$.

To achieve the desired purpose, Narma L-2 is employed. As it can be seen in the figs. 5 and 6, both the DC motor input and the cart position hit their respective constraints.

Figure 5. Narma L-2 based inverted Pendulum Response for angle $\theta$ and control force $u$.
Figure 6. Narma L-2 based inverted Pendulum Response for angle $\theta$ and position $x$

CONCLUSION

In this paper the application of NARMA L2 controller on the inverted pendulum is discussed. NARMA L-2 controller I here is a discrete-time controller which contains the internal feedback of past control effort and output, designed to omit both of the nonlinearity and the dynamic behavior of the nonlinear inverted pendulum system.

NARMA L-2 can transform the considered nonlinear system into an implicit algebraic model easily and efficient control of the trajectory. Experimental results show that the proposed NARMA L2 controller can control the nonlinear inverted pendulum to follow the smooth desired trajectory.

REFERENCES


