International Research Journal of Applied and Basic Sciences

ISSN 2251-838X / Vol, 4 (1): 205-212
Science Explorer Publications

Vibration analysis of piezoelectric nanowires using the finite element method

M. Haghpanahi[1], A. Oveisi[1]*, M. Gudarzi[2]

1. School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran
2. Department of Mechanical Engineering, Damavand branch, Islami Azad University, Damavand, Iran

****Corresponding Author email:*** a_oveisi@mecheng.iust.ac.ir

ABSTRACT: A finite element model for free vibrations of piezoelectric nanowires (NWs) is developed based on the non-local Euler-Bernoulli beam theory and Galerkin method. The governing equation of motion for the piezoelectric NW with consideration of non-local effects and surface effect is obtained, and the approximate expressions for the natural frequencies and the fundamental buckling voltage are derived for simply supported boundary conditions. Also, an explicit relationship between the residual surface tension and the small scale parameter of the piezoelectric NW, based on the critical electric potential is offered at which the axial buckling occurs. Consequently, a method is given that may assist in experimental application of NW based instruments.
Keywords: Vibration; Nano-wires; Finite element; Galerkin method; Surface effect; Nonlocal theory; Euler Bernoulli.

INTRODUCTION

NWs have been among the most studied nanomaterials in recent years. They hold great promise for many technologically critical applications as sensors, actuators, transistors, probes and resonators in NEMS/MEMS devices and biotechnology (Wu et al., 2001; Cui et al., 2003). In order to consider the size effect, two different approaches have been proposed one is based on the high order continuum model for constituent materials; the other argues that the interface effect comes into play (Chen et al., 2007). In particular, among the size-dependent continuum theories, the theory of non-local continuum mechanics initiated by Eringen (1972), Eringen and Edelen (1972). A wide study on lattice dispersion of elastic waves, wave propagation in composites, dislocation mechanics, fracture mechanics, surface tension fluids, etc have been done (Lu et al., 2006; Wang et al., 2006; Wang, 2005; Sudak, 2003; Reddy and Wang, 1998; Peddieson et al., 2003). Also, Peddieson et al. (2003) applied non-local elasticity to formulate a non-local version of the Euler-Bernoulli beam model and concluded that non-local continuum mechanics could potentially play a useful role in nanotechnology applications. Surface effects are significant to nanostructure materials and can influence the physical and chemical properties of nanomaterials due to the increasing ratio of surface area to volume. Therefore, much effort has been performed to investigate the surface effects on nanostructures (Wang and Feng, 2009; He and Lilley, 2008; Hasheminejad and Gheshlaghi, 2010). For example, (Gurtin et al., 1976) investigated the effect of surface elasticity on the resonance frequency of nanobeams. Miller and Shenoy (2000) considered the stretching and bending problems of nanosized structural elements. Also, the impact of residual surface stress and surface elasticity on the vibration and buckling of nanobeams through the Laplace-Young equation is addressed (Wang and Feng, 2009; Wang and Feng, 2009). Chen et al. (2007) formulated a theoretical framework to examine the size effect due to both non-local and interface effects for composite materials and found that these effects dominate the size-dependent effective property of the material on nanoscale. NWs with piezoelectric properties are being developed for converting nanoscale mechanical energy into electric energy (Song et al., 2006; Gao et al., 2007; Wang and Song, 2006). As the dimension of various devices is reduced down to micro/nano scale, the need for nanoscale piezoelectric energy producing devices is consequently getting greater in order to effectively power various nanosystems (Song et al., 2006). Recently, direct electricity generation from piezoelectric NWs has been successfully demonstrated (Su et al., 2007; Wang et al., 2007; Song et al., 2006). In these applications, the mechanical deformation, induced by the deflection of cantilevered NW with a probe tip or ultrasonic wave, produces an electric response which is sensed through the probe tip. Also, Wang and Feng (2010) studied the surface effects on the vibration and buckling of the piezoelectric NWs. Much effort has been done in order to investigate the macroscopic piezoelectric materials and devices (Gopinathan et al., 2000; Milazzo et al., 2009). Study of the size-dependent behavior of piezoelectric NWs in presence of the surface effects and based on the theory of non-local elasticity has not been reported in the literature. These effects can become more and more pronounced with decreasing the size of the piezoelectric NWs (Wang and Feng, 2009; Reddy and Wang, 1998). Accordingly, in this paper, the non-local elasticity theory integrated with surface elasticity model (Chen et al., 2007) will be employed to study the free vibrations of non-conducting piezoelectric Euler-Bernoulli NWs using FEM. Also, a buckling analysis is conducted in order to obtain a theoretical model for describing the relationship between the surface and the non-local parameter of the piezoelectric NW.

Formulation

Figure 1 shows a schematic diagram of the geometry of the considered problem. The NW of length L, width b and thickness $2h$ is assumed to be simply supported between a pair of substrates coated by thin, continuous layers of electrodes. Because of the thin configuration of the electroded substrates the dynamic characteristics of the NW does not alter significantly.

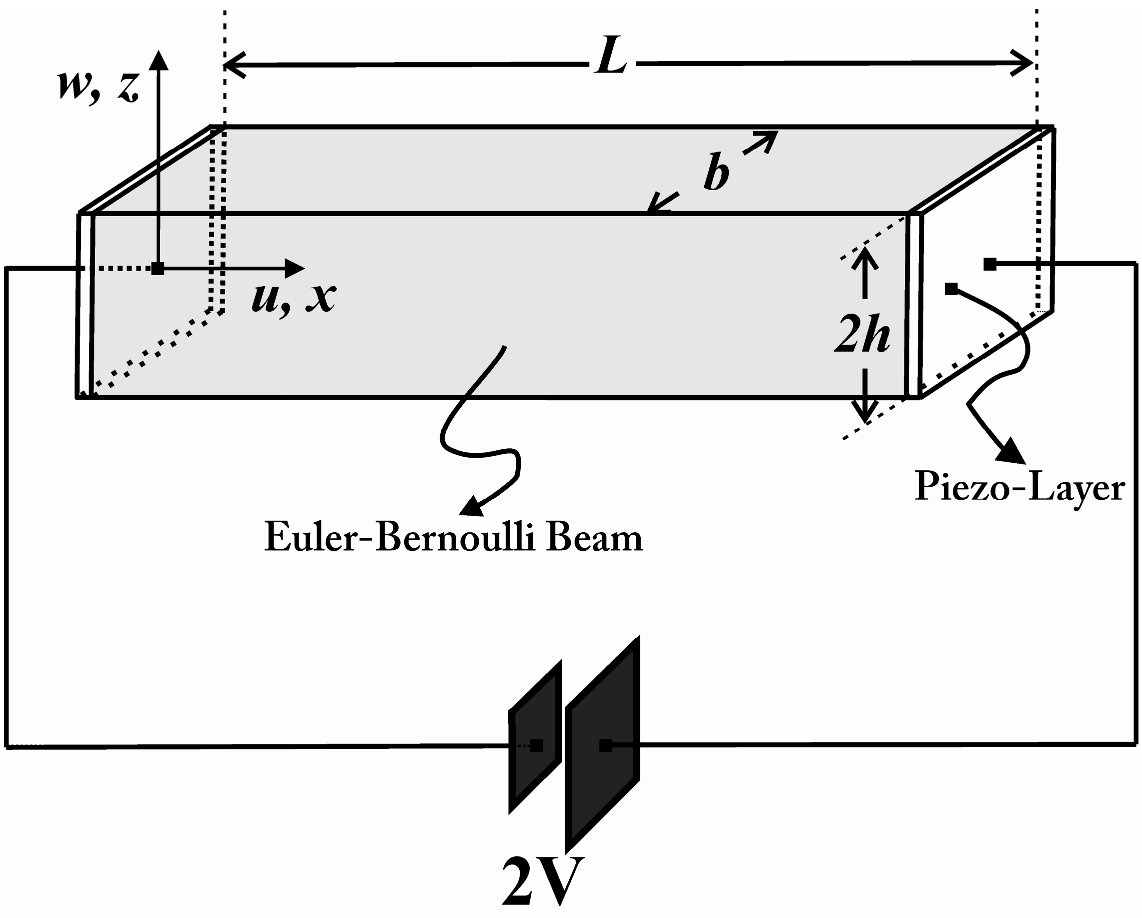

Figure 1. The problem geometry.

For a brief discussion on the relevancy of present model in regard to realistic devices, the reader is referred to Cha et al. (2010). Also, the axial displacement u and the transverse displacement w are respectively expressed as

$$u = -z\frac{\partial w}{\partial x}, \qquad w = w(x,t). \tag{1}$$

where t is the time. In this study both of the surface and the small scale effects are taken into account simultaneously (Chen et al., 2007). The inclusion of the surface effects starts by considering uniform surface layers of infinitesimal thickness on the top and the bottom surfaces of the NW (Baron et al., 2010). Accordingly, the normal stress, σ_x and the surface stress, σ_s are related to the strain, ε_x through the following linear elastic constitutive relations

$$\sigma_x = E\varepsilon_x - e_{31}\Phi_z, \qquad \sigma_s = \tau^0 + E^s\varepsilon_x, \qquad \varepsilon_x = -z\frac{\partial^2 w(x,t)}{\partial x^2} \tag{2}$$

where E and e_{31} are the linear elastic Young's modulus and the piezoelectric coefficient, respectively, τ^0 is the residual surface tension and E^s is the surface Young's modulus. Also, the electric-field components Φ_x and Φ_z are related to the electric potential ϕ through the following relations

$$\Phi_x = -\frac{\partial\phi}{\partial x}, \qquad \Phi_z = -\frac{\partial\phi}{\partial z}, \tag{3}$$

Recent calculations show that the piezoelectric potential in a bending piezoelectric NW is nearly independent of the axial coordination along the NW, except in the vicinity of two ends (Gao and Wang, 2007). In particular, it was found that the deflection of the piezo NW by application of an electric potential $\phi(x,z)$ across the width of the NW (i.e., through the converse piezoelectric effect; (Cady, 1946)), creates a strain field, with the outer surface being stretched (positive strain) and the inner surface being compressed (negative strain). Thus, the electric potential varies between $\phi(x,-h)$ to $\phi(x,h)$ across the width of the NW from the compressed to the stretched side surfaces, and one may assume a uniform piezoelectric potential distribution along the NW (x-axis), implying that $\Phi_x \ll \Phi_z$ (Wang and Song, 2006).

In the absence of electric charges, the electrostatic equilibrium condition can be expressed as

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} = 0, \tag{4}$$

where D_x and D_z are the electric displacement components, given by Gao and Wang (2007) as

$$D_z = e_{31}\varepsilon_x + \lambda_{33}\Phi_z, \qquad D_x = \lambda_{11}\Phi_x \tag{5}$$

where λ_{11} and λ_{33} are dielectric constants. As λ_{11} and λ_{33} are on the same order, with consideration of $\Phi_x \ll \Phi_z$, thus, one can neglect the electric displacement D_x in comparison with D_z. Substitution of Eqs. (2), (3), and (5) into (4), while assuming $\phi(x,-h) = \phi(x,h) = 2V$ (see Fig. 1), the electric potential ϕ is obtained as (Gopinathan et al., 2000; Milazzo et al., 2009)

$$\phi(x,z) = -\frac{e_{31}}{\lambda_{33}}\left(\frac{z^2 - h^2}{2}\right)\frac{\partial^2 w(x,t)}{\partial x^2} + \left(1 + \frac{z}{h}\right)V, \tag{6}$$

Additionally, the equation of transverse vibrations for a beam based on the Euler-Bernoulli beam theory is expressed by

$$\frac{\partial^2 M}{\partial x^2} = \rho A\frac{\partial^2 w(x,t)}{\partial t^2} - \frac{d}{dx}\left(P\frac{\partial w(x,t)}{\partial x}\right), \tag{7}$$

where A and ρ denote the (uniform) cross sectional area and the mass density, respectively, $P = 2b\tau^0 + P_e(x,t)$ is the total axial force, with $P_e(x,t) = b\int_{-h}^{h}\sigma_x\, dz = 2Vbe_{32}$ being the electric potential induced component, and M is the non-local moment, which has the following special form for NWs (Reddy, 2008; Wang and Feng, 2009)

$$M(x,t) - \mu\frac{\partial^2 M(x,t)}{\partial x^2} = -(EI)_{eff}\frac{\partial^2 w(x,t)}{\partial x^2} \tag{8}$$

where

$$(EI)_{eff} = 2E^s bh^2 + \frac{4E^s}{3}h^3 + b\int_{-h}^{h}\left(1 + \frac{e_{31}^2}{E\lambda_{33}}\right)Ez^2 dz \tag{9}$$

In which $(EI)_{eff}$ is effective bending stiffness, and μ is the small scale parameter (Reddy, 2008). Direct substitution of Eq. (7) into Eq. (8), leads to the final expression for the non-local bending moment

$$M(x,t) = -(EI)_{eff}\frac{\partial^4 w(x,t)}{\partial x^4} + \mu\left[\rho A\frac{\partial^2 w(x,t)}{\partial t^2} - \frac{d}{dx}\left(P\frac{\partial w(x,t)}{\partial x}\right)\right]. \tag{10}$$

Therefore, by substituting (10) in (7), the governing equation of transverse vibration for the beam with consideration of both surface and nonlocal effects can be expressed as

$$(EI)_{eff}\frac{\partial^4 w(x,t)}{\partial x^4}+\left[1-\mu\frac{\partial^2}{\partial x^2}\right]\left[\rho A\frac{\partial^2 w(x,t)}{\partial t^2}-\frac{d}{dx}\left(P\frac{\partial w(x,t)}{\partial x}\right)\right]=0 \quad (11)$$

Now, following the standard free vibration analysis procedure, the analytical solutions for fee vibrations of piezoelectric NWs under axial load P, is sought by assuming periodic solutions of the form $w(x,t)=W(x)e^{-i\omega t}$, where $W(x)$ is the mode shape and ω is the frequency of vibration. Substitution of this latter assumption into Eq. (11) casts the governing equation in the form

$$\left\{((EI)_{eff}+\mu P)\frac{d^4W(x)}{dx^4}+(P+\mu\rho A\omega^2)\frac{d^2W(x)}{dx^2}-\rho A\omega^2 W(x)\right\}e^{-i\omega t}=0 \quad (12)$$

The Galerkin method is the most widely used weighted residual method. In this method, solution of the eigenvalue problem is assumed in the form of a series of n comparison function which satisfy all boundary conditions of the problem

$$W(x)=\sum_{i=1}^{n}c_i\psi_i(x) \quad (13)$$

where c_i are coefficients to be determined and $\psi_i(x)$ are known comparison functions. By substituting Eq. (13) into Eq. (12) one can obtain

$$((EI)_{eff}+\mu P)\sum_{i=1}^{n}c_i\frac{d^4\psi_i(x)}{dx^4}+(P+\mu\rho A\omega^2)\sum_{i=1}^{n}c_i\frac{d^2\psi_i(x)}{dx^2}-\rho A\omega^2\sum_{i=1}^{n}c_i\psi_i(x)=R(c_i,x) \quad (14)$$

where $R(c_i,x)$ is the resulting error or residual function. In order to obtain a series of linear equations that leads to natural frequency equation one should multiply Eq. (14) by $\psi_j(x)$, and use the orthogonality of mode shapes as

$$\int_0^L R(c_i,x)\psi_j(x)dx=0 \quad (15)$$

$$\sum_{i=1}^{n}c_i\left[((EI)_{eff}+\mu P)\int_0^l\frac{d^4\psi_i(x)}{dx^4}\psi_j(x)dx+(P+\mu\rho A\omega^2)\int_0^l\frac{d^2\psi_i(x)}{dx^2}\psi_j(x)dx-\rho A\omega^2\int_0^l\psi_i(x)\psi_j(x)dx\right]=0 \quad (16)$$

Defining the symmetric stiffness, damping, and mass coefficients k_{ij} and m_{ij} respectively, as

$$k_{ij}=((EI)_{eff}+\mu P)\int_0^l\frac{d^4\psi_i(x)}{dx^4}\psi_j(x)dx+P\int_0^l\frac{d^2\psi_i(x)}{dx^2}\psi_j(x)dx \quad (17a)$$

$$m_{ij}=\int_0^l\rho A\psi_i(x)\psi_j(x)dx+\mu\rho A\int_0^l\frac{d^2\psi_i(x)}{dx^2}\psi_j(x)dx \quad (17b)$$

Using Eqs. (17) , the equation Eq. (16) can be rewritten as

$$\omega^2\sum_{i=1}^{n}c_i m_{ij}+\sum_{i=1}^{n}c_i k_{ij}=0 \quad (18)$$

Or in the matrix form as

$$\omega^2[M]\vec{c}+[K]\vec{c}=0 \quad (19)$$

In order to find the natural frequencies, one should find the roots of following matrix equation

$$|\omega^2[M]+[K]|=0 \quad (20)$$

In which $|\cdot|$ denotes the determinant of the corresponding argument. By using of comparison functions as $\psi_i(x)=\sin\left(\frac{i\pi x}{L}\right)$ which satisfies the simply support boundary condition at two ends of the NW, the natural frequency of the

NW can be calculated using Galerkin method. In order to validate the results achieved by the finite element method with the previous studies the solution of eigenvalue problem which is given in Eq. (13) is assumed as

$$W(x) = c\psi(x) \quad (21)$$

And the frequencies are obtained as

$$\omega_1 = \left(\frac{\pi}{L}\right)\sqrt{\frac{L^2 P + (\pi)^2[((EI)_{eff} + \mu P)]}{\rho A(L^2 + \mu\pi^2)}}, \quad (22)$$

Also By setting ω_1 zero, one can obtain the expression for fundamental buckling voltage (i.e., $n = 1$) in the form

$$V_b = -\frac{(EI)_{eff}(\pi)^2}{2e_{31}(L^2 + b\mu(\pi)^2)} - \frac{\tau^0}{e_{31}}, \quad (23)$$

where V_b is the buckling voltage. Furthermore, using the above expression, the following useful relation between the residual surface tension and the non-local parameters of a piezoelectric NW is obtained

$$\tau^0 = -e_{31}V_b - \frac{(EI)_{eff}(\pi)^2}{2e_{31}(L^2 + b\mu(\pi)^2)} \quad (24)$$

It can be seen from Eq. (15) that the residual surface tension τ^0 has an upper bound equal to $-e_{31}V_b$. Also by ignoring the non-local effects, one will have

$$\tau^0 = -e_{31}V_b - \frac{(EI)_{eff}(\pi)^2}{2bL^2} \quad (25)$$

RESULTS

Some numerical examples are considered in order to examine both the surface and small scale effects for a simply supported piezoelectric NW of square cross section with the following physical properties (Gao and Wang, 2007),

$$\frac{L}{2h} = 20\ (-h \le z \le h), \qquad E = 207\ \text{Gpa}, \qquad \rho = 7800\ {}^{\text{kg}}/_{\text{m}^3},$$
$$e_{31} = -0.51\ {}^{\text{C}}/_{\text{m}^2}, \qquad \lambda_{33} = -7.88 \times 10^{-11}\ {}^{\text{F}}/_{\text{m}}$$

For crystalline metals, atomic simulations display that τ^0 and E^s are on the same order (Shenoy, 2005). Furthermore, in the case of linear elastic deformation, the contribution of surface elasticity to the total surface stresses can be neglected in comparison with residual surface stress. The experimental values of surface constants τ^0 and E^s for piezoelectric materials are not available in the literature, we shall not consider the surface elasticity effects in our numerical examples, and the residual surface stress constant is assumed to be $\tau^0 = 0.5\ \text{N/m}$ along with two selected values for the small scale parameter.

Figure 2 displays variation of the normalized fundamental natural frequency of the piezoelectric NW with its length for elected input voltages and non-local parameters.

Here, the natural frequency is normalized with respect to the fundamental frequency calculated using the classical Euler- Bernoulli beam model i.e., neglecting the surface and small scale parameters as well as the piezoelectricity effects as

$$\omega_n = (n\pi)^2(EI)_{eff}/(\rho AL^4) \quad (26)$$

Here, it is clear that by increasing the NW length, both surface and small scale effects gradually disappear. Also, decreasing the input voltage leads to an expected overall decrease in the calculated NW natural. Furthermore, one can note that by including the non-local effects for very short NWs under a negative input voltage ($L < 50\ nm, V = -1\ \text{volt}, \mu = 10^{-14}\text{m}^2$), the natural frequency is calculated to be zero, i.e., harmonic motion would not be possible for the piezoelectric NW, and buckling may occur. Moreover, one should note that the calculated natural frequency of the piezoelectric NW without the non-local effects (i.e., $\mu = 0\ \text{m}^2$), perfectly match the numerical results obtained using Eq. (17) in (Wang and Feng, 2010) (also, see Eq. (13)). The most interesting observation is perhaps the fact that including the non-local effects (i.e., $\mu = 10^{-14}\text{m}^2$), causes a notable drop in the value of NW natural frequencies (i.e., like a damping effect), nearly regardless of input voltage. Lastly, by making use of Eq. (15), one can obtain a useful design chart describing the relationship between the residual surface tension and the non-local parameter of the piezoelectric NW.

This is done in Fig. 3 for a piezoelectric NW of length $L = 200\ nm$, for selected values of buckling electric voltages. Finally, Fig. 4 shows the Variation of residual surface tension with respect to beam length, applied voltage and small scale parameter.

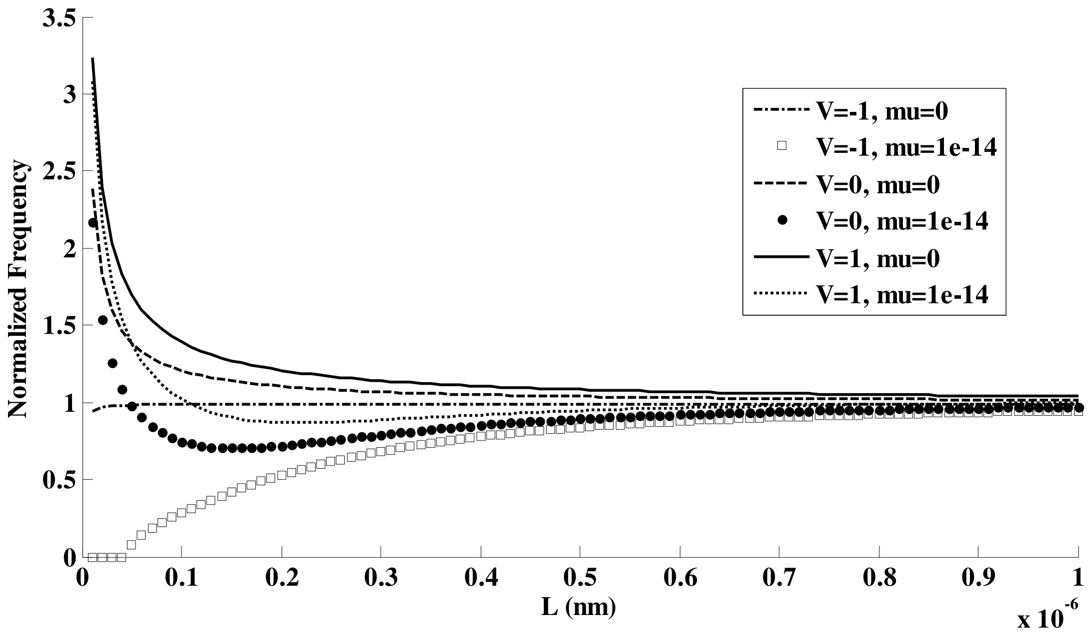

Figure 2. Variation of the normalized fundamental natural frequency with NW length for selected input voltages and non-local parameters

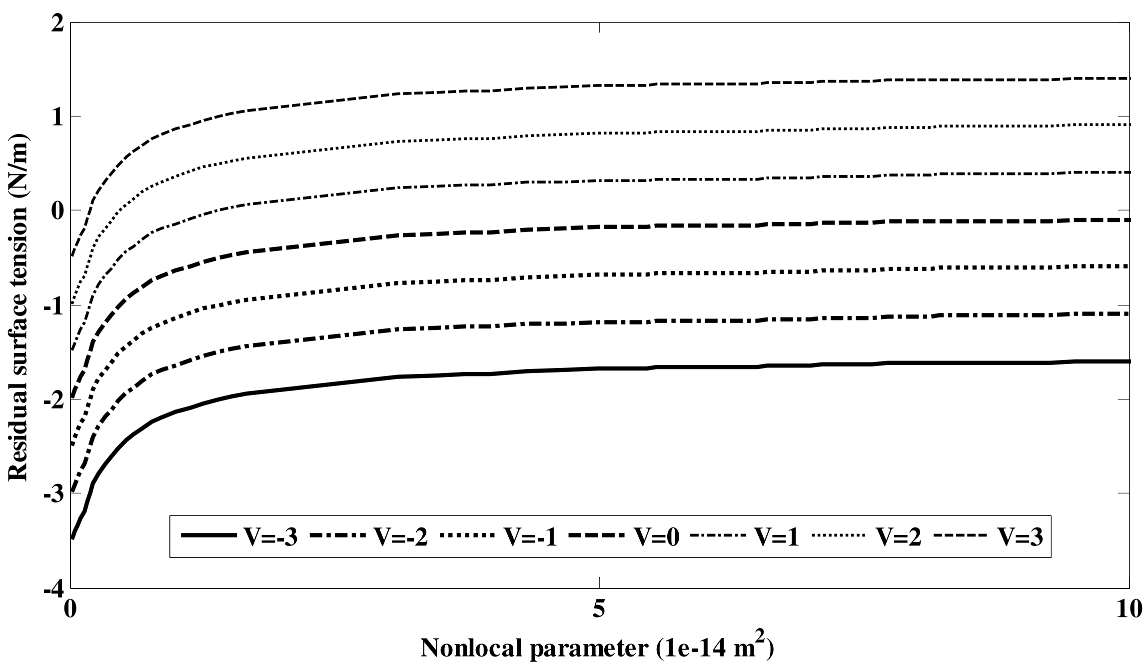

Figure 3. Design chart describing the relationship between the residual surface tension and the non-local parameter of the piezoelectric NW for selected values of buckling electric voltage

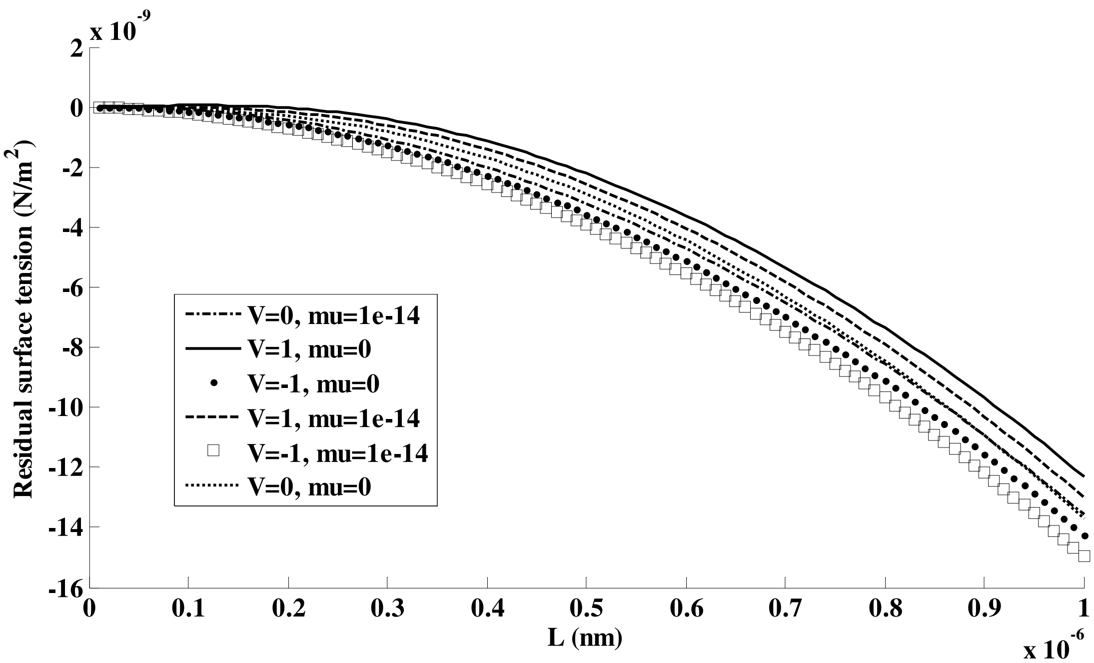

Figure 4. Variation of residual surface tension with respect to beam length, applied voltage and small scale parameter

CONCLUSIONS

This paper makes the first attempt to study the free vibration of piezoelectric Euler-Bernoulli NWs in presence of both the surface and the non-local elasticity effects using the FE method. Furthermore, a theoretical criterion for describing the relationship between the residual surface tension and the small scale parameter of the piezoelectric NWs is presented. This information can complement the experimental measurement of the critical electric potential at which the axial buckling occurs. It is seen that the resonant frequency of piezoelectric NW can be tuned by adjusting the applied electric potential. Also, the small scale parameter (non-local elasticity effect) can significantly affect the predicted resonant frequency of piezoelectric NW. Furthermore, an explicit expression (design chart) between the residual surface tension and the non-local parameter is presented, which can aid in experimental characterization of piezoelectric NWs.

REFERENCES

Baron B, Altus E, Tadmor EB. 2010. Surface effects in non-uniform nanobeams: continuum vs. Atomistic modeling, Int. J. Solids Struct. 47: 1243-1252.

Cady WG. 1946. Piezoelectriciry, McGraw-Hill Book Company Inc., New York.

Cha SN, Seo JS, Kim SM, Kim HJ, Park YJ, Kim SW, Kim JM. 2010. Sound-driven piezoelectric nanowire based nanogenerators. Adv. Mater. 22: 4726-4730.

Chen H, Gengkai H, Zhuping H. 2007. Effective moduli for micropolar composite with interface effect. Int. J. Solids Struct. 44: 8106-8118.

Cui Y, Zhong ZH, Wang DL, Wang WU, Lieber CM. 2003. High performance silicon nanowire field effect transistors. Nano Lett. 3: 149-152.

Eringen AC, Edelen DGB. 1972. On non-local elasticity. Int. J. Eng. Sci. 10: 233-248.

Eringen AC. 1972. On non-local fluid mechanics. Int. J. Eng. Sci. 10: 561-575.

Gao PX, Song JH, Liu J, Wang ZL. 2007. Nanowire piezoelectric nanogenerators on plastic substrates as flexible power sources for nanodevices. Adv. Mater. 19: 67-72.

Gao YF, Wang ZL. 2007. Electrostatic potential in a bent piezoelectric nanowire. The fundamental theory of nanogenerator and nanopiezotronics. Nano Lett. 7: 2499-2505.

Gopinathan SV, Varadan VV, Karadan VK. 2000. A review and critique of theories for piezoelectric laminates. Smart Mater. Struct. 9: 24-28.

Gurtin ME, Markenscoff X, Thurston RN. 1976. Effect of surface stress on the natural frequency of thin crystals. Appl. Phys. Lett. 29: 529-530.

Hasheminejad SM, Gheshlaghi B. 2010. Dissipative surface stress effects on free vibrations of nanowires, Appl. Phys. Lett. 97: 2531031-2531033.

He J, Lilley CM, 2008. Surface effect on the elastic behavior of static bending nanowires. Nano Lett. 8: 1798-1802.

Lu P, Lee HP, Lu C, Zhang PQ. 2006. Dynamic properties of flexural beams using a nonlocal elasticity model. J. Appl. Phys. 99: 073510-073518.

Milazzo A, Orlando C, Alaimo A. 2009. Analytical solution for the magneto-electroelastic bimorph beam forced vibrations problem. Smart Mater. Struct. 18: 085012-085018.

Miller RE, Shenoy VB. 2000. Size-dependent elastic properties of nanosized structural elements. Nanotechnology. 11: 139-147.
Peddieson J, Buchanan GG, McNitt RP. 2003. Application of nonlocal continuum models to nanotechnology. Int. J. Eng. Sci. 41: 305-312.
Reddy JN, 2008. Nonlocal continuum theories of beams for the analysis of carbon nanotubes. J. Appl. Phys. 103: 023511-023526.
Reddy JN, Wang CM. 1998. Deflection relationships between classical and third order plate theories Acta Mech. 130: 199-208.
Shenoy VB. 2005. Size dependence of thermal expansion of nanostructures. Phys. Rev. B. 71: 0941041-0941044.
Song JH, Zhou J, Wang ZL. 2006. Piezoelectric and semiconducting coupled power generating process of a single ZnO belt/wire. A technology for harvesting electricity from the environment. Nano Lett. 6: 1656-1662.
Su WS, Chen YF, Hsiao CL, Tu LW. 2007. Generation of electricity in GaN nanorods induced by piezoelectric effect. Appl. Phys. Lett. 90: 0631101-0631103.
Sudak LJ. 2003. Column buckling of multiwalled carbon nanotubes using nonlocal continuum mechanics. J. Appl. Phys. 94: 7281-7287.
Wang CM, Zhang YY, Ramesh SS, Kitipornchai S. 2006. Buckling analysis of micro- and nano-rods/tubes based on nonlocal Timoshenko beam theory. J. Phys. D: Appl. Phys. 39: 3904-3909.
Wang GF, Feng XQ. 2009. Influence of surface energy on the stiffness of nanospring. Appl. Phys. Lett. 94: 1419131-1419133.
Wang GF, Feng XQ. 2009. Timoshenko beam model for buckling and vibration of nanowires with surface effects. J. Phys. D. 42: 155411-155415.
Wang GF, Feng XQ. 2010. Effect of surface stresses on the vibration and buckling of piezoelectric nanowires. EPL. 91: 1-4.
Wang Q. 2005. Wave propagation in carbon nanotubes via nonlocal continuum mechanics. J. Appl. Phys. 98: 124301-124306.
Wang XD, Song JH, Liu J, Wang ZL. 2007. Direct-current nanogenerator driven by ultrasonic waves. Science. 316: 102-105.
Wang ZL, Song JH. 2006. Piezoelectric nanogenerators based on Zinc Oxide nanowire arrays. Science. 312: 242-246.
Wu G, Ji H, Hansen K, Thundat T, Datar R, Cote R, Hagan MF, Chakraborty AK, Majumdar A. 2001. Origin of nanomechanical cantilever motion generated from biomolecular interactions. A Proc. Natl. Acad. Sci. USA. 98: 1560-1564.