A method for Inter-Area Power Systems Stability

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ABSTRACT: There are special characteristic of the stability behavior for the inter-area power systems. Therefore, the improvement of inter-area power system is one of the important aspects in power system. To reduce the detrimental effects of inter-area oscillations on system stability, this research uses Linear Matrix Inequalities (LMIs) to design a multi-objective state feedback. This paper presents a detailed analysis of low-frequency inter-area oscillations of systems. The influences of component maintenances in the transmission corridors on the inter-area oscillations are further investigated, and some important conclusions are drawn. We find a control law that stabilizes several contingencies simultaneously using a polytopic model of the system. We used the technique on machines and get satisfactory results.

Keywords: Inter-Area, Power Systems, Stability, Polytopic, Linear Matrix Inequalities

INTRODUCTION

Inter-area oscillations ranging from 0.1 to 0.7 Hz are common phenomena in power systems. To analyze these oscillations, two basic methods have been developed. One is eigenvalue analysis, which has been effective and widely used. The models of the entire system are linearized at the operating point, and eigenvalues are computed from this linearized state matrix. Among the most commonly used techniques, the modified Arnoldi method (MAM) is shown to be the most reliable in analyzing very large power systems (Snyder A F al, 1999). Prony analysis, the other one, decomposes time-domain signals into damped sinusoids with four parameters per mode: frequency, damping, amplitude and phase (Snyder A al, 1999). For this method, large amount of computation on Eigen values is not required, and it is suitable for field measurement analysis. Prony analysis and Eigen analysis are complementary methods in the study of low-frequency inter-area oscillations.

Because of growing demand in electrical energy, Modern power system network is getting much more complicated than ever before. Some power systems are linked between two area “inter-area power systems”. The inter-area power systems have special characteristic of stability behavior. The improvement of inter-area power system is one of the important aspects in power system. There are many advanced devices have been proposed during the last three decades to improve stability of power system such as High Voltage Direct Current (HVDC) system and Flexible AC Transmission System (FACTS) devices (Anamitra P al, 2004) and (Scherer CW al, 1995).

Since the 1920’s, low frequency electromechanical oscillations have been problematic for power grids across the world. These small oscillations from .1-1 Hz are an anticipated phenomenon which occurs in the grid (Chan-Ki al, 2009). They can develop between groups of machines on a network (Kosterev D N al, 1999). Therefore, the power engineer must generate solutions to damp and control these low frequency oscillations before they cause more serious problems in the system.

First, the cause of these oscillations needs to be identified. In early grid development, the system was designed to allow for growth. As the system grew rapidly, the transmission network became increasingly stressed and the grid was working closer to transient and small-signal rotor angle stability limits. Insufficient synchronizing torque became a serious cause for system instability. During the mid-century, utilities aimed to increase transient stability with high response exciters. However, in an effort to aid in the first swing transient stability, the low frequency oscillations were exacerbated (Rogers GJ, 2000).

These high response exciters create problems for the lower frequency local modes. As the network strength decreases, the exciters can lead to further steady state instability. The first swing would be stable, but then would become unstable. Additionally, because the load was growing faster than anticipated, generator groups began to form. The increase in demand for power weakened the tie lines connecting these generator groups.

The electrical length of the tie lines and the inertia of the machines determine the frequency of the oscillations (Rogers G, 2000). In one instance, in a larger interconnection in Canada, the oscillations would
generally go undamped until an impedance relay was tripped resulting in the loss of a line (Hui Ni al, 2002). A system operating with lightly damped inter-area modes is stressed (Camino JF al, 2004).

Even in the earlier studies of inter-area oscillations in the 1960s, it was concluded that it was important for dynamic stability problems to be approached as a system wide problem. Synchronous generator excitation and speed control needed to be adjusted based on information from the whole system. Since then, the power industry has advanced greatly, and it has become much easier to design controls which can evaluate the system as a whole.

New developments in remote wide area phasor measurements (WAMS) provide an opportunity for damping these oscillations. Both local and remote measurements can be taken with phasor measurement units (PMUs). These signals are sampled and time synchronized with a precision of one microsecond (Camino JF al, 2004). Thus, WAMS allow for the synchronized transfer of data across the power network (Machowski J al, 1997). This means that oscillations can be detected in real-time. Additionally, they can be controlled in real-time with whatever type of control that is chosen or be co-ordinated with a control that is already in place (Rugh J, 1996). Therefore, WAMS are an invaluable tool in the control of the modern grid. Additionally, studies have shown that using WAMS to enhance existing controllers is more cost effective than installing new devices (Mekki K al, 2001).

**Literature review**

The control solutions mentioned above can be implemented with many control techniques. However, for this paper, linear matrix inequalities (LMIs) were chosen as a way to utilize HVDC lines. The advantage of using LMI is to control the system is that they work for numerous contingencies. This is more appealing than traditional control design, which focuses on developing a control for each individual disturbance, because it is more time and cost effective in addition to being more robust. Furthermore, the LMI control used in MATLAB Robust Control Toolbox can define a multi-objective problem in which the MATLAB program solves for the optimum solution for H2/H∞ norms based on a damping region the user defines. The disturbance rejection is attained with the H∞ control, the control effort is optimized with H2 control, and the poles are placed with a desired minimum damping ratio (Scherer CW al, 1995). The time delays are represented in the system as uncertain parameters. The control aims to ensure feasibility of the solution, linear objective minimization, and eigenvalue minimization (L’Abbate A al, 2010).

Consider the power system, shown in Fig. 1, as in Refs (Bouhamida M al, 2005). The basic configuration includes two areas which are connected through long double AC transmission lines, two local loads, and a controllable SVC connected at the midpoint of the transmission line to control the power flow and to increase the system’s damping. The SVC also includes a fixed shunt capacitive compensator.

![Figure 1. Inter-area power transmission system.](image)

The dynamic behavior of the primary mode of the system, as shown in Fig. 1, can be described by the swing equation, as given in Refs (Chilali M al, 1996) and (Klein M al, 1991).

Where \( \omega \) is the relative phase velocity and \( \delta \) is the relative phase angle between the two areas, which are respectively defined as \( \omega = \omega_1 - \omega_2 \) and \( \delta = \delta_1 - \delta_2 \). Also, function \( f \) and the powers \( P_1c \) and \( P_2c \), delivered to the system by areas 1 and 2, are given by

\[
f = \frac{P_{m1} - P_{1L}}{M_1} - \frac{P_{m2} - P_{2L}}{M_2} + \frac{D_1}{M_1} \omega + \frac{D_2}{M_2} \omega_2,
\]

\[
a_i = \frac{X_i}{R_i^2 + X_i^2}, \quad \beta_i = \frac{X_i}{R_i^2 + X_i^2} \text{ for } i = 1, 2
\]

Where

Note that since the reactance of a transmission line is much greater than its resistance, and since the damping coefficients are very small, it may be assumed that \( R_1, R_2, D_1 \) and \( D_2 \) are zero (Dorato P al, 2000). This would simplify the above equations without noticeably affecting the accuracy of the transient response.

In the above system, under steady-state conditions, the two areas operate synchronously. The relative rotor speed is zero, and the phase angles of the two areas remain unchanged. However, in the case of a line outage (fault), the system’s structure changes and creates power flow imbalance in the transmission lines,
which leads to power oscillation. In this case, for power flow control and transient stability, the power oscillation must be controlled very quickly. Sliding controllers have the capability for fast and continuous adjustment of the reactive power QSC, to enhance the system’s damping, by rapidly adjusting the impedance of the SVC or, in effect, by modulating the line voltage Vc at the midpoint. The controller design, however, must take into consideration the physical constraints imposed on the system. That is, first, the SVCs typically have low power ratings. This, in effect, puts a bound on the magnitude of the reactive power compensation QSC, generated by the SVC. Also, generally, the SVCs cannot inject real power into the system. That is, the real power Psc, generated by the SVC, is zero at all times. In addition, the magnitude of the line voltages Vc, which is modulated by the SVC, must stay within a permissible range at all times. Satisfying these constraints at all times increases the complexity of the controller design.

Let us now discuss the different types of power system oscillations. The types of oscillations are determined by the part of the power system affected by them. The electromechanical oscillations are studied particularly because they affect small signal stability. The following is a list of several types of electromechanical oscillations (Gahinet P al, 1996):

- Intra-plant mode oscillations
  - Torsional modes between rotating plants
    - X These are much higher frequency modes in the range of 10-46 Hz which are associated with the turbine generator shaft. These can occur when a multi-stage turbine generator is connected to the grid through a series compensated line.
    - X Local modes are caused by one generator swinging against the rest of the system at 0.7-2.0 Hz (So PL al, 2005). This oscillation is local to the generator and the line which connects it to the grid, thus allowing the system to be modeled normally. These oscillations can be compensated for with a PSS which modulates the voltage reference of the AVR (Khargonekar PP al, 1991).
  - Control mode oscillations
    - X These modes are a side effect of controls placed in the system. They can include poorly tuned SVCs, HVDC converters, excitors and governors.
    - X Inter-area mode oscillations. Inter-area modes are caused by two groups of generators in a network swinging against each other at a frequency of about 2.0-3.0 Hz.
    - X These oscillations generally occur in large interconnected systems. Very low frequency modes in the range of 0.1-0.3 Hz arise because of all of the system’s generators. The other type occurs when the system splits into groups, and the generators in one group swing against another group. These oscillations occur at a slightly higher frequency of approximately 0.4-1.0 Hz (McGraw-Hill, 2006). This oscillation is local to the generator and the line which connects it to the grid, thus allowing the system to be modeled normally. These oscillations can be compensated for with a PSS which modulates the voltage reference of the AVR (Khargonekar PP al, 1991).

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    - X These are much higher frequency modes in the range of 10-46 Hz which are associated with the turbine generator shaft. These can occur when a multi-stage turbine generator is connected to the grid through a series compensated line.
  - Control mode oscillations
    - X These modes are a side effect of controls placed in the system. They can include poorly tuned SVCs, HVDC converters, excitors and governors.
  - Inter-area mode oscillations. Inter-area modes are caused by two groups of generators in a network swinging against each other at a frequency of 1 Hz or less (So PL al, 2005). They will be discussed in greater detail below. The inter-area modes, which are of particular interest for this paper, are described in more detail below. As previously stated, inter-area modes are caused by groups of generators in a network swinging against each other at a frequency of 1 Hz or less. The damping of inter-area modes is determined by several factors. These include the tie-line strength, the loads, the power flow through the lines, and the interactions of generators and their controls (Kalman RE al, 1961). It is easy to see how these types of oscillations have become an issue. Additionally, all of the different types of factors which contribute to inter-area oscillations make them difficult to study and understand. In many instances, it is necessary to have a system model to study an inter-area mode (Saribulut L al, 2009).
  - Two types of inter-area oscillations generally occur in large interconnected systems. Very low frequency modes in the range of 0.1-0.3 Hz arise because of all of the system’s generators. The other type occurs when the system splits into groups, and the generators in one group swing against another group. These oscillations occur at a slightly higher frequency of approximately 0.4-1.0 Hz (McGraw-Hill, 2006).
  - In order to study these modes, it is best to evaluate a small system. The small four-machine, two-area system developed in (Zhang Y al, 2008) provides several ways to analyze these modes.
    - Because many of the components in a power system are characteristically explained by differential algebraic non-linear equations, the computational complexity associated with them is high. These include generators, excitation systems, governors, and loads. For a small system, the non-linear control techniques can be applied, but when the system grows, it is not practical to solve such a problem.
    - However, for the purposes of this paper, linear control theory can provide useful information about the system. When low frequency oscillations occur due to small disturbances, they are approximately linear (Kalman RE al, 1961). A small disturbance is also one in which the system dynamics can be linearized. The differences in machine angles and speeds are small when disturbances such as small fluctuations of generation and load occur. The differential algebraic non-linear equations can be linearized with respect to a system equilibrium point. For more information about the linearized system equation.
    - It is now necessary to evaluate the requirements of a controller in power systems. A robust control system is one which has little to no sensitivity to the difference between the actual system and the model used for designing the control. Robustness requires the control to provide adequate damping and a security margin during all operating conditions (Snyder A al, 1999).
  - 2. Based on utility guidelines, all oscillations must settle within a particular time.
  - 3. The robustness measure of performance and stability margins require that the damping must not decrease to unacceptable levels during varying operating conditions and network configurations.
4. There should be no adverse interaction between controllers for different devices (i.e. they should be coordinated through multivariable control design). Additionally, the control system must include performance objectives which make the system respond in a desirable manner. The rise time, settling time, steady state offset, gain and phase margins are indications of performance. These can be characterized in either the time domain or the frequency domain. Many different types of power system controllers exist. The following is a brief introduction into several of the more common varieties.

There are several classical controls which are used in power systems. These include Automatic Voltage Regulators (AVRs) and Power System Stabilizers (PSSs).

An AVR is used in local control elements. It can regulate the generator terminal voltage through control of the amount of current supplied to the generator field winding by the exciter. In order to regulate the field current and the exciter output, the voltage measured at the generator terminal is compensated for by the load current. The generator terminal voltage is compared to the desired voltage reference. The difference in the generator terminal voltage and the desired reference voltage is what alters the field current and exciter output. Therefore, the difference is decreased to zero. It is a closed-loop system (Hui N et al, 2002).

A PSS control is one of the traditional forms of controls currently in use in Power Systems. This device is added to an AVR loop to improve damping during power swings. It provides a component of the electrical torque in the synchronous machine rotor which is proportional to the deviation of the actual speed from the synchronous speed. This means that when the rotor oscillates, the torque damps the oscillation (Snyder A et al, 1999). The commonly used stabilizing signals include shaft speed, terminal frequency, and power (Hui N et al, 2002). Although PSSs are common forms of control, they are mostly used for damping local modes. They can actually have a negative effect on inter-area modes if used inappropriately (Zhang Y et al, 2008). An example of an inappropriate use would be when the PSS sees an oscillation and operates, even though that mode is not actually a problem for the system. If it could see the whole system, the PSS would not apply its control and would not damp that mode. The negative effects of the PSS can be mitigated by integrating wide area measurements into the system and only acting when the mode is detrimental to the system as a whole (Zhang Y et al, 2008) and (Saribulut L et al, 2009).

Flexible AC Transmission Systems (FACTS) devices developed from thyristor based technologies help to improve system stability. They are solid state designs that are being applied to power systems to increase stability. They can provide fast, continuous control over the power flows in a system such that generators and load shedding may not be needed to maintain system stability (Snyder A et al, 1999). Additionally, they provide a way to change the power flows so that they are optimal for the equipment and the economic dispatch is obeyed (Snyder A et al, 1999). SVCs – Static VAR Compensators (SVCs) have been utilized since the 1970s, even though FACTS devices were not developed at that time. The basic principle behind SVCs is to provide reactive power compensation when the system is in need. It includes both capacitive and inductive elements that can be introduced quickly to adjust to rapidly changing loads. The generic SVC is composed of a thyristor controlled reactor and a fixed capacitor. TCSC – A Thyristor Controlled Series Capacitor (TCSC) is another common FACTS device. It is a capacitive reactance compensator consisting of a series capacitor bank shunted by a thyristor controlled reactor. The goal is to smooth the variations in series capacitive reactance (Snyder A et al, 1999). This is done by changing the firing angles which changes its apparent reactance.

ESD – An Energy Storage Device (ESD) can help to stabilize and improve the reliability of the power system. Examples of these include flywheels, advanced capacitors, and battery energy storage systems. Rather than supplying reactive power, these devices can provide real power to the system rapidly and without detrimentally impacting power flow. There is research currently being done at Virginia Tech about the placement of these devices in power systems (Zhang Y et al, 2008).

Coordinated Controls – Although each individual device can provide some form of control in a power system, the benefits of combining these controls are being further explored. With multiple controls in place, damping all of the inter-area modes becomes a more realistic goal. Coordinated control is not possible without Wide Area Measurements (WAMS) which are provided by Phasor Measurement Units (PMUs).

There are several papers which provide information on their studies of coordinated control. In (Zhang Y et al, 2008), the coordinated control of TCSCs and SVCs was developed for increasing damping of inter-area oscillations with successful results. (Bouhamida M et al, 2005) and (Saribulut L et al, 2009) also explore different types of coordinated controllers.

The development of HVDC (High Voltage Direct Current) transmission systems was facilitated in the 1930s with the invention of mercury arc rectifiers. In 1941, work on HVDC transmission systems had begun in Germany. However, because of WWII, no system actually was implemented. In 1954 the first HVDC transmission system came to fruition in Gotland, a large island province of Sweden. By the 1960s, HVDC transmission systems had evolved into a mature technology.

These systems can play a vital part in both long distance transmission and in the interconnection of systems. HVDC transmission systems combine high reliability with a long useful life. They must include a
power converter to interact with the AC transmission system. Conversion back and forth between AC and DC occurs using controllable electronic switches (valves) in a 3-phase bridge configuration. Generic HVDC converters have capacitors at both ends for reactive power compensation. There is real power input at one end and it exits to a load at the other end.

Additionally, HVDC lines are advantageous because they do not have problems that are associated with AC lines, namely (Rogers G, 2000):

- No length limit
- No synchronism requirement
- No increase to short circuit capacity imposed on AC switchgears
- Not affected by impedance, phase angle, frequency, voltage changes
- Improves AC system reliability, thereby increasing carrying capacity by modulating power in response to power swings or frequency fluctuation

Unlike AC lines, the transmission on HVDC lines is not limited by reactive power constraints (Rugh J, 1996). They cannot become overloaded because the power flow through them is controlled, meaning that they do not have to be sized to handle a contingency reserve (Rugh J, 1996).

Economics plays a major role in the selection and use of HVDC technology (Gahinet P al, 1996). The cost of a DC transmission line varies from 80% to 100% of the cost of an AC line with the same rated line voltage. However, over long distances, DC transmission may be rated at twice the power flow capacity of an AC line of the same voltage. AC lines no longer become a viable option for transmission over long distances underground or under water on a technical basis. Capacitive charging current associated with AC is the reason for this. Additionally, there are several environmental benefits to using DC lines. These can be separated into two categories. One set of effects is those associated directly with the flow of current in the power lines. A second set of effects consists of those caused by the mere presence of power lines in the environment.

Effects arising from the presence of power in transmission lines may be separated into field and ion effects and corona effects. Power lines produce both electric fields and charged airborne particles. There has been concern that either the fields or the charged particles emitted at low levels may cause detrimental health effects. However, epidemiology studies do not generally support these concerns.

The second set of environmental issues arises in connection with the mere siting of power lines in the outdoor environment. Because power lines are large physical structures which occupy a lot of space, people in the communities which host them believe they negatively impact the atmosphere. These range from aesthetic concerns, negative effects on property values in their vicinity, obstruction of view, and cultural issues to transportation hazard, wetlands impacts, deforestation, and harm to water resources. The comparative footprint of AC and DC power lines merits discussion. For the same amounts of power transmission, DC power lines occupy less land than AC lines. Taking less land means less cost for transmission line construction. It may mean less public opposition, as well.

The information given is just to provide a small introduction to HVDC lines and why they are a viable option for power system control. It is important to evaluate the different aspects of a technology before studying it to ensure that it is not only for the intellectual pursuit, but also for practical purposes.

**MATERIALS AND METHODS**

The test system was developed by taking the IEEE 16 Machine Model system and dividing it into the New England portion. The entire 16 machine model can be found in (Klein M al, 1991). This smaller model has eight classically modeled generators and 15 buses with one HVDC line. The generators are classically modeled with the rotor angle and generator speeds. The system will be discussed in further detail in the following sections.

**Basic Mathematical Concepts**

Before the state space system can be determined, the load flow must be performed. For this work, the Graham Rogers PSTV3 MATLAB Suite was used. It utilizes a Newton-Raphson algorithm to solve for the load flow. Next, it is desirable to evaluate the small signal stability. Small signal stability is the stability of an operating point of a dynamic system when perturbed with small disturbances. The system behavior in the small frequency range evaluated here is generally expressed as a set of non-linear differential and algebraic equations. These algebraic equations come from the stator current equations (Snyder A al, 1999). The initial operating points are obtained by substituting the algebraic variables into the differential equations. Then, the set of equations is linearized about an equilibrium point. More information on this can be found in (Snyder A al, 1999) and (Hui Ni al, 2002).

Because this is a small signal analysis, the state space equations are an essential part of analyzing a power system. There are several ways to linearized a system, including using Newton's method and calculating the Jacobian matrix or using the small perturbations method and calculating manually. The PSTV3 Suite was
used to find the state matrices by numerically perturbing each state of the model. Next, the program took the
change (or difference) in the state and divided it by the magnitude of the perturbation. When the perturbed
values return to equilibrium, the new values are returned to their initial condition. This process is then repeated,
and thus provides the generic linear time invariant equations expressed by (5.1).

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}
\]

Where
A is the state matrix. It is square and has the dimension equal to the number of states.
B is the input matrix. It gives the proportion of the individual inputs that are applied to each state equation.
C is the output matrix.
D is the feed forward matrix.
x(t) is the state vector.
u(t) is the input vector.
y(t) is the output vector.

**Eigenvalues and what they indicate**

Whether real parts of all eigenvalues in a system are negative is important. When they are, transients
in a linear dynamic system decay over time. A stable system is one in which all transients decay. An unstable
system is one in which one or more transients grow. This is a problem for power systems because when
unstable, they will not operate properly. However, because power systems are nonlinear, a linear system model
must be created. A linearized model may have a complex eigenvalue with a positive real component. When this
happens, system oscillations of constant amplitude can occur, subject to limitation by the system’s nonlinearity.
The system, though compromised, can remain in operation for at least a while. However, a growing oscillation
can result in system collapse and failure. The system is threatened in either case.

At a basic level, the first step made towards controlling the system needs to be evaluating the role each
generator plays in each mode. The first step is looking at the solution to the state equation.

\[
X = \sum_{i=1}^{n} u_i z_i
\]

- \(u_i\) is the \(i^{th}\) eigenvector of A
- \(z_i\) is the \(i^{th}\) mode

One way to evaluate the roles of the generators is through modal analysis. When a state vector of a
particular mode has a large entry corresponding to its right eigenvector, it should be evaluated (L’Abbate A al,
2010) and (Snyder A al, 1999). This eigenvector is commonly referred to as the mode shape. Another way of
stating this is that a mode’s right eigenvector provides the relative amplitude of the mode which is observed
through the dynamic system states (Klein M al, 1991). The largest amplitude of oscillation for a particular mode
corresponds to the state with the largest eigenvalue magnitude. When an eigenvector coefficient is zero for a
certain state, the measurements of that state cannot see that mode (Klein M al, 1991). The mode shapes can
determine coherent machine groups in multi-machine systems (Snyder A al, 1999). The \(i^{th}\) eigenvalue, \(i^{th}\) left
eigenvector, and system input define the \(i^{th}\) mode as a scalar function of time.

The next way to determine the effectiveness of controls on inter-area oscillations is to look at a
compass plot of the rotor angle terms in the state matrix. This right eigenvector is used to evaluate the state
changes that occur when that mode of oscillation is excited. Therefore, the plot will show the modes oscillating
against one another. It is an indication of the ability to monitor the mode from particular states, but it does not
necessarily indicate if they are good for controlling them. This is why the participation factor and compass plot
are used together (Klein M al, 1991).

**Controllability and Observability**

The controllability of a system is how a mode will respond when the inputs are perturbed. When a
mode responds to an input change, the mode is controllable through that input. The observability of a system
describes how each mode is seen in the system outputs. When a mode is seen by a measurement at the
system output, the system is called observable in that output. There are several tests to evaluate the
observability of the system. Controllability and observability are dual notions of the system (Khargonekar PP al,
1991). These concepts can be studied in more detail in (Khargonekar PP al, 1991).
RESULT AND DISCUSSION

Feasibility is the first component of this system that must be tested. For the requirements of the adaptive control testing, the controller must not be able to damp all five contingencies with a single control, i.e., there is no feasible solution to that problem. As expected, the control did not damp all five cases. Thus, the five contingencies were broken into two polytopes with the common base case. However, for this system, the controller was split into different polytopes. The polytopes actually split into the groups seen in Fig. 2.

Now, it is necessary to solve for the controllers $k_1$ and $k_2$ for each polytope. There are several design specifications that surround this control choice.

The damping region chose was about 5%. This is much smaller than what is desirable, but with only one control in a relatively stressed, small system, it is satisfactory. The gain matrix, $K$, could provide damping up to about 10%. However, for future demonstrations, I decided they should keep the same damping region. The $H_2/H_{\infty}$ trade-off found that there was no consequential benefit to spending time finding the Pareto-optimal point. This may be more desirable if the system was larger, but under these circumstances, weighting their importance equally is also satisfactory. The following section presents the graphs of the LMI testing. The first graph, Fig. 1, shows the open and closed loop poles of $P_1$.

In Figure 3, there are the open and closed loop poles of $P_2$. The controller of the second polytope was able to make some of the poles move even further to the left. However, this was at the expense of controller cost.
Figure 4 is the root locus plot for S2 with K1. The root locus plot shows the direction the poles are moving when the control is applied. They behave correctly in this example because all of them move to the left. The squares represent the initial position of the eigenvalues whereas the circles denote their final position. From the figure, it becomes clear that all the eigenvalues have moved towards the left implying that the proposed adaptive selection method has been successful in choosing the correct control. The next step is to test this algorithm. To do so, I formed the polytopes previously seen in Fig. 3. The first polytope, P1, was tested by assuming that the input vector is composed of equally weighted components of the vertices of P1.

\[
x(m) = \frac{1}{3}(x_1(m) + x_2(m) + x_4(m))
\]

Therefore, when the \( \theta_1, \theta_2, \theta_4 \) matrices are used to find \( x(m) \) the vector is defined by (5.17), the scalar values of \( a_1, a_3, and a_4 \) should be constant at the value of \( 1/3 \). There could be a small initial perturbation as the recursive algorithm begins, but it should settle to a constant value quickly. The graph of the \( a' \) for this test is shown in Fig. 5. Note that if the input value \( m(x) \) were weighted differently, but still such that the weights summed to one, the \( a' \) would take those values instead. This is more clearly explained in a modification to (5.17). \( m(x) = 0.2x_1(m) + 0.2x_2(m) + 0.6x_4(m) \) the corresponding steady-state vector would now be, \( a = [0.2 0.2 0.6]^t \).
Next, it was necessary to ensure that the algorithm returned incorrect values of \( a \) when an incorrect \( m(x) \) is chosen. The values of the F1 matrix are kept the same. The input vertices which compose are changed as seen in.

\[
x(m) = \frac{1}{3} (x_1(m) + x_2(m) + x_5(m))
\]

The vector is now composed of values representative of the vertices of P2. Though the weighting is still the same, the vertices provided cannot be controlled by the same controller as P1. The resulting \( a_1, a_2 \) and \( a_5 \) are exactly as predicted. They do not form a convex combination with the polytopic variables, and thus do not make a convex set. Fig. 6 shows how sporadic these values are.

![Graph showing alpha values for input chosen in P1](image)
CONCLUSION

The control techniques explored in this paper offer a broad range of tools for tackling a very large and long standing problem: inter-area oscillations. The LMI poly topic formulation allows for many different and opposite constraints to be fulfilled while still finding a single sub-optimal controller. The various control devices also offer solutions for the problems at hand. Many of these FACTs based devices are becoming more popular, less expensive, and more common. Their popularity will continue because of these factors. Additionally, utilizing FACTs devices to increase stability has an economic and environmental component too. Using them is an economically sound way to increase stability, at least in the shorter term. Also, it is more environmentally conservative to utilize this technology than build more transmission lines. This is the point where we, as engineers, enter the picture. This paper provides a small contribution towards the ultimate goal of power system stability. Even though this paper develops an algorithm in a theoretical world, it may one day be developed into something which could be applied to a real network. Twenty years ago digital relays were considered inconsistent and impossible to implement, but one would find it difficult to locate a working electromechanical relay in a substation now.

There are many constraints to take into consideration if this theory will ever be truly applied to a real power system. There are issues of redundancy of signal transfer similar to that in protection schemes. What signals in the system are so important that they should be redundant would need to be determined. There would need to be testing of the information signals as well. Testing would also need to indicate if the firing angles used in the DC implementation are correct. In power system protection, a relay can be removed from service for extensive testing. How can this system be checked if it cannot be removed? Additionally, if a mode were to occur, does the utility manager rely on the control algorithm to work without his or her input? The generally conservative nature of the power systems industry would indicate that an operator would see a mode and put the correct type of control into place. However, this could be adapted into a completely automated system.

REFERENCES

Anamitra P, James S. Thorp, “Co-ordinated control of inter-area oscillations using SMA and LMI”, accepted for publication in IEEE PES Conference on Innovative Smart Grid Technologies


