Design LQR and PID Controller for Optimal Load Sharing of an Electrical Microgrid

M. Lotfollahzade, S.J. Seyed-Shenava, A. Akbarimajd, J. Javidan

University of Mohaghegh Ardabili, Ardabil, Iran

Corresponding Author email: jshenava@gmail.com

ABSTRACT: Interconnection of distributed generators (DGs) to the utility grid through power electronic converters has raised concern about proper load sharing between different distributed generators and the grid. In this paper control strategies for proper load sharing between utility and parallel converter connected in a microgrid and supplied by DG are described. The DG has local loads that are very close to it. These loads can be unbalanced and/or non-linear. A control method is proposed to load balancing and proper load sharing in both islanded and grid-connected modes. This method helps DG to compensate the effects of unbalance and non-linearity of the local loads using LQR controller and then, using a novel PSO algorithm designs a PID controller for proper load sharing between utility and DG. Both grid-connected and island mode responses are considered. The efficiency of the controller has been verified through simulations for various operating Conditions Using Matlab-Simulink.

Keywords: Droop Characteristics, Islanding, Particle Swarm Optimization, State Feedback Controller, Voltage Source Converter

INTRODUCTION

In this paper microgrid viewed as cluster of DGs connected to the main utility grid, through some voltage source converter (VSC)-based interfaces. In general, The DG may have local loads that are very close to it. Common loads are also connected to the microgrid, which are supplied by the utility grid under normal conditions. An important issue, concerning interconnection of a microgrid to the utility system, is to study the overall system performance with unbalanced and non-linear loads. In this paper a scheme for controlling parallel connected converters in both islanded and grid-connected modes is presented. The most common technique of local load sharing is the droop characteristics. Parallel converters have been controlled to deliver desired real and reactive power to the system. Local signals are used as feedback to control the converters. The real and reactive power sharing can be achieved by regulating two autonomous quantities – the power angle and the fundamental voltage magnitude (Guerrero J.M et al, 2004; Reza. M et al, 2006; Chandorkar M.C et al, 1993). Different microgrid control strategy and power management techniques are discussed by (Sao C.K et al, 2008; Nikkhajoei H et al, 2009). The system stability during load sharing has been explored by many researchers (Guerrero J.M et al, 2004; Chandorkar M.C et al, 1993). The main aim of this paper is to set up power electronics interfaced microgrid containing DGs. The proposed controller will be able to cancel out the effect of load nonlinearity using state feedback control and shares local loads properly using PID controller.

MATERIALS AND METHODS

System structure

Single line diagram of the basic system structure considered in this paper is shown in Fig.1. It can be seen that it contains one DG unit. The microgrid is connected to the utility grid at PCC. In normal condition, DG shares its local load with the utility, whereas the common load is supplied entirely by the utility. During islanding, DG supplies both its local load and common load, totally. The complex power drawn by the local loads is $P_{L1} + jQ_{L1}$. The common load draws a current $i_{LC}$ and a complex power of $P_{LC} + jQ_{LC}$. 
Reference current generation scheme

Compensator reference current generation in grid-connected mode

In this subsection, it is assumed there is no common load (i.e. CB-2 is open), while DG-1 is only supplies part of its local load. In this paper, the reference current generation method proposed in (Majumder R et al, 2009; Ghosh A et al, 2000) is utilized. Based on this method we then get the following equation for generating reference current from the measured circuit variables.

\[
\begin{bmatrix}
i_{1a} \\
i_{1b} \\
i_{1c}
\end{bmatrix} = \begin{bmatrix}
i_{L1a} \\
i_{L1b} \\
i_{L1c}
\end{bmatrix} - \frac{1}{K} \begin{bmatrix}
3P_{G1}v_{pla} + \sqrt{3}Q_{G1}(v_{plb} - v_{plc}) \\
3P_{G1}v_{plb} + \sqrt{3}Q_{G1}(v_{plc} - v_{pla}) \\
3P_{G1}v_{plc} + \sqrt{3}Q_{G1}(v_{pla} - v_{plb})
\end{bmatrix}
\]

(1)

where: \( K = 3 \left( v_{pla}^2 + v_{plb}^2 + v_{plc}^2 \right) \). \( P_G \) and \( Q_G \) are the instantaneous real and reactive powers respectively.

Then we can write:

\[
v_{pla}i_{g1a} + v_{plb}i_{g1b} + v_{plc}i_{g1c} = P_{G1}
\]

(2)

Similarly, the reactive powers \( Q_G \) will be equal to its instantaneous component, that is:

\[
(v_{plb} - v_{plc})i_{g1a} + (v_{plc} - v_{pla})i_{g1b} + (v_{pla} - v_{plb})i_{g1c} = \sqrt{3}Q_{G1}
\]

(3)

As it is mentioned earlier, DG-1 supplies a fraction of the average real and reactive power demanded by the local load, then, we can write

\[
P_1 = \lambda_{1P} \times P_{L1av}, \quad Q_1 = \lambda_{1Q} \times Q_{L1av}
\]

(4)

\[
P_{G1} = P_{L1av} - \lambda_{1P} \times P_{L1av} = P_{L1av}(1 - \lambda_{1P})
\]

\[
Q_{G1} = Q_{L1av} - \lambda_{1Q} \times Q_{L1av} = Q_{L1av}(1 - \lambda_{1Q})
\]

(5)

Then, using (1), (5) we can write the following reference currents for \( i_1 \).
In the above algorithm, terms $P_{Lav}$ and $Q_{Lav}$ are obtained by a moving average filter that gives a continuous measurement of the average power by averaging it over the immediate previous half cycle.

**Compensator reference current generation in island mode**

When an islanding occurs, the DG will have to supply both its local load and common load totally. Therefore in islanding mode, we have $P_{Lav} = Q_{Lav} = 1$. Moreover $P_{G1}$, $Q_{G1}$, and $i_{g1}$ will be negative with respect to the directions shown in Figure.1. The generalized form for the reference currents of DG-1 can be given from (1) and (6) as (Majumder R et al. 2009):

$$
\begin{bmatrix}
i_{la} \\
i_{lb} \\
i_{lc}
\end{bmatrix} =
\begin{bmatrix}
i_{lla} \\
i_{llb} \\
i_{llc}
\end{bmatrix} - \frac{1}{K} \begin{bmatrix}
3P_{Lav}(1 - \lambda_{tp})v_{p} + \sqrt{3}Q_{Lav}(1 - \lambda_{tq})(v_{p} - v_{l}) \\
3P_{Lav}(1 - \lambda_{tp})v_{p} + \sqrt{3}Q_{Lav}(1 - \lambda_{tq})(v_{p} - v_{l}) \\
3P_{Lav}(1 - \lambda_{tp})v_{p} + \sqrt{3}Q_{Lav}(1 - \lambda_{tq})(v_{p} - v_{l})
\end{bmatrix}
$$

(6)

In above equation, $\lambda_{tp}$ and $\lambda_{tq}$ are, respectively, the common load real and reactive power demand fractions supplied by utility. In island mode, $\lambda_{tp} = \lambda_{tq} = 1$. However, in the grid-connected mode, when the local load is shared by the DG-1 and the grid we have $0 < \lambda_{tp} \leq 1$, $0 < \lambda_{tq} \leq 1$ and $\lambda_{tp} = \lambda_{tq} = 1$.

**Converter structure**

Structure of VSCs is shown in figure 2. It includes three H-bridge converters those are connected to ideal dc source. Each VSC is connected to the network through a transformer. The purpose of using the transformers is to provide isolation between the converters (Ghosh A et al, 2000). The inductance $L_f$ in this figure represents the leakage inductance of each transformer and additional external inductance, if any. The switching losses of an inverter and the copper loss of the connecting transformer are represented by a resistance $R_f$. The iron losses of the transformer are neglected. The filter capacitor $C_f$ is connected to the output of the transformers to bypass switching harmonics. The impedance $L_1$ is added to provide output impedance of the DG source.

**Converter control**

The equivalent circuit of one phase of the converter is shown in Figure 3.b. In this figure $u.v_{dc1}$ represents the converter output voltage, where $u=\pm 1$. The main purpose of the converter control is to generate $u$ (Majumder R, 2010).

**RESULTS**

**Simulation studies**

Simulation studies are carried out in MATLAB SIMULINK. Both grid-connected and island mode responses are considered. Parameters of LQR performance index are selected as: $Q = \text{diag}[0 \ 1 \ 1]$ and $R=0.05$.

Two variables that are most important to be controlled are the current $i_k$ and the voltage $V_{PCC}$. The weighting matrix $Q$ represents the importance of these states.

Example 1. (grid connected mode). Consider the system shown in Figure 1. it’s assumed that microgrid is connected to utility, hence, the local load shared by the DG and utility, and DG do not share the common load. In this case, the current references of DG compensator are calculated from (6). The system data are given in Table 1. It is desired that DG-1 shares 10% of the real power and 20% of the reactive power of its on local load. So, in this case, $\lambda_{tp}=0.1$, $\lambda_{tq}=0.2$. Assume at $t=0.5$ s, the common load impedance is made half of its initial value. Figure.5 shows Power sharing of DG-1. The voltages of PCC1 and the current $i_{g1}$ are shown in...
Figure 6. The power sharing in the same desired ratio and balanced voltages even after change in the common load shows a stable operation.

From Figure 3.b, state space equation of the system can be given as

\[ x_1 = Ax_1 + Bu_c + Cv_{PCC} \]  

(8)

Where \( u_c \) is the continuous time version of switching function \( u \). The discrete-time equivalent of (8) is:

\[ x(n+1) = Fx(n) + Gu_c(n) + Hv_{PCC}(n) \]  

(9)

Based on this model and suitable feedback control law, \( u_c(n) \) is computed. The equivalent circuit of one phase of the converter is shown in figure 4.

From this circuit of the following state vector is chosen:

\[ x_1^T = \begin{bmatrix} i_1 & i_2 & v_{cf} \end{bmatrix} \]  

(10)
State feedback controller

In state feedback controller, states of the system are compared with their reference values to generate the converter switching. Generating references for the output voltage $v_{cf}$ and current $i_2$ can be easily done by power flow condition, however, it is not straightforward for reference of $i_1$ (Majumder R et al, 2010). To address this issue, the following state transformation can be helpful:

$$ x^T = \begin{bmatrix} i_c & i_2 & v_{cf} \end{bmatrix} $$  \hspace{1cm} (11)

The transformed state space equation is then given as:

$$ \dot{x} = C_p x + D_p u $$  \hspace{1cm} (12)

where $C_p$ is state transformation matrix.

The state-space equation of the circuit then can be written as:

$$ \begin{bmatrix} i_1 \\ i_2 \\ v_{cf} \end{bmatrix} = \begin{bmatrix} \frac{1}{L_f} & 0 & -1 \\ 0 & \frac{1}{L_f} & 0 \\ \frac{1}{C_f} & \frac{1}{C_f} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_{cf} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{uV_{dc}}{L_f} \end{bmatrix} v_{PCC} $$  \hspace{1cm} (13)

The control law is then given by:

$$ u_c(n) = -K \left[ x_1(n) - x_{ref}(n) \right] $$  \hspace{1cm} (14)

where $K$ is a gain matrix and $x_{ref}$ is the reference vector. The gain matrix, in this paper is obtained through linear quadratic regulator design with a state weighting matrix $Q$ and a control penalty $R$. In an LQR problem, a performance index of the form

$$ j = \int_0^\infty (x-x_{ref})^T Q(x-x_{ref}) + u^T Ru \, dt $$  \hspace{1cm} (15)

is chosen. It is then minimized to obtain the optimal control law $u$ through the solution of steady state Riccati equation (Anderson B.D.O et al, 1971).

From $u_c(n)$, the switching function is generated as

$$ u_c(n) > h \quad \text{then} \quad u = +1 $$

$$ u_c(n) < -h \quad \text{then} \quad u = -1 $$  \hspace{1cm} (16)

where $h$ is a small number and called hysteresis band (Ghosh A, Ledwich G, 2003).

Computation of Reference vectors:

To implement the switching control using the state feedback controller, the references for compensator current of DG-1 is computed using equation (7). The reference voltage is set from the positive sequence fundamental component of the PCC1 voltage. Therefore the positive sequence of the fundamental of the PCC1 voltage is extracted from (Ghosh A, Ledwich G et al., 2003). Reference for the current $i_{cf}$ is computed by phase shifting the reference for $V_{PCC1}$ by 90º and from the knowledge of the value of the filter capacitor $C_f$. 

Figure 4. Single phase equivalent circuit of VSC
The voltage of PCC1 is shown in Fig.6. In this figure the voltage drops at t=0.5 s clearly seen. After a few seconds (about 0.025 s) the voltage balanced again. Table 2 shows some characteristics of these figures.

Example 2. (Islanding mode). Consider the same system as given in example 1. Suppose in t=0.4 (s) CB-1 opens and an islanding occurs where the common load remains connected. It is desired now that both local load and common load are totally supplied by the individual DG. Fig. 7.a shows the real power sharing of DG-1. The voltage of PCC1 is shown in Fig. 7.b. At the onset of the islanding, PCC1 voltage drop, causing a slight drop in P1. However, P1 increase to supply the common load, as indicated by negative power flow in P_G1. At 1.0 s, the utility is reconnected and about at t=1.17 s, the power sharing goes back to the initial values. When the utility reconnected, at t=1.0 s, power sharing have some overshoot and undershoot and system balancing take about 0.17 seconds. The numerical results of these figures are shown in table 3. In the next example for better performance of the network and power sharing, we design a PID controller in feedback loop.

### Table 1. System parameters

<table>
<thead>
<tr>
<th>System quantities</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source voltage (Vs)</td>
<td>11 KV rms (L-L)</td>
</tr>
<tr>
<td>Systems frequency</td>
<td>50HZ</td>
</tr>
<tr>
<td>Feeder impedance</td>
<td>( R_s = 1.025 \Omega, L_s = 57.75mH )</td>
</tr>
<tr>
<td>Common impedance load</td>
<td>( R_{La} = 24.4 \Omega, L_{La} = 100.0mH )</td>
</tr>
<tr>
<td>DG-1 local unbalanced load</td>
<td>( R_{La} = 48.4 \Omega, L_{La} = 192.6mH )</td>
</tr>
<tr>
<td>DG-1 local nonlinear load</td>
<td>a three-phase rectifier supplying an RL load with R = 200( \Omega ), L = 100mH</td>
</tr>
<tr>
<td>Microgrid line impedance</td>
<td>( R_{D1} = 0.2 \Omega )</td>
</tr>
<tr>
<td>DG and compensator</td>
<td>3.5 KV</td>
</tr>
<tr>
<td>DC voltage (( V_{dc1} ))</td>
<td>3KV/11KV, 0.5 MVA, 2.5% reactance (Lc)</td>
</tr>
<tr>
<td>Transformer rating</td>
<td>1.5 ( \Omega )</td>
</tr>
<tr>
<td>VSC losses</td>
<td>50 ( \mu F )</td>
</tr>
</tbody>
</table>
Table 2. Numerical results for sharing the local load with utility (Example 1)

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Active power</td>
<td>Initial value</td>
<td>Final value</td>
<td>Min value</td>
<td>Max undershoot</td>
<td></td>
</tr>
<tr>
<td>$P_{L1}$</td>
<td>0.7</td>
<td>0.7</td>
<td>0.56</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.628</td>
<td>0.63</td>
<td>0.44</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>reactive power</td>
<td>Initial value</td>
<td>Final value</td>
<td>Min value</td>
<td>Max undershoot</td>
<td></td>
</tr>
<tr>
<td>$Q_{L1}$</td>
<td>0.76</td>
<td>0.76</td>
<td>0.62</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>$Q_1$</td>
<td>0.61</td>
<td>0.61</td>
<td>0.5</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>$Q_{G1}$</td>
<td>0.15</td>
<td>0.15</td>
<td>0.12</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Numerical results for islanding mode (Example 2)

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Active power</td>
<td>Initial value</td>
<td>Intermediate value</td>
<td>Final value</td>
<td>Reconstruction time (s)</td>
<td></td>
</tr>
<tr>
<td>$P_{L1}$</td>
<td>0.7</td>
<td>-0.13</td>
<td>0.625</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.629</td>
<td>0.76</td>
<td>0.07</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>$P_{G1}$</td>
<td>0.071</td>
<td>5</td>
<td>4.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 3: Design PID controller in feedback loop using PSO algorithm

Consider the same system as given in example 2. In that example, we see that after reconnection, the power sharing have some overshoot and undershoot and system balancing take about 0.17 s. we can improve transient operation of the system with designing PID controller at state feedback controller.

\[
\dot{x}_i = A x_i + B_i u_i + \Gamma V_{nc} \\
\]

PID controller

\[
0.01 \leq K_p, K_i, K_d \leq 2
\]

The block diagram of controller is shown in figure 8 and PID coefficients are calculate using standard particle swarm optimization method (Standard PSO), (J. Kennedy, R.C. Eberhart 1995- E. Elbeltagi et al 2005).

The objective function which should be minimized is defined as below:

\[
F = \int_{t_0}^{t_{fs}} \sum_{i} \left| P_i - P_{1ss} \right| dt, i = L_1, G_1, 1
\]

where $P_{1ss}$, $P_{1ss}$, and $P_{G1ss}$ are the final value given in table 3. In order to acquire better performance, number of particle, particle size, maximum iteration cycles, $c_1$ and $c_2$ is chosen as 30, 3, 30, 0.1 and 0.4.
respectively. Also, the inertia weight, \( w \), is linearly decreasing from 1 to 0.3. The final values of the optimized parameters using standard PSO techniques are given in Table 4. The real power sharing of DG-1 and voltage of PCC1 are shown in figure.9.a and figure.9.b respectively. It can be seen from these figures, the system after reconnection balanced, quickly and the system responses has good performance comparing example2. Numerical results for this state shown in table 5.

Table 4. Optimized parameters of PID controller

<table>
<thead>
<tr>
<th>( K_p )</th>
<th>( K_i )</th>
<th>( K_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7970</td>
<td>1.4167</td>
<td>0.0100</td>
</tr>
</tbody>
</table>

Figure 9a. Real power sharing of DG-1 (with PID controller) (MW)

Figure 9b. Three phase Voltages at the PCC1

Table 5. Numerical results for islanding mode (with PID controller) (Example 3)

<table>
<thead>
<tr>
<th>Active</th>
<th>Initial value (MW)</th>
<th>Intermediate value (in islanded mode), MW</th>
<th>Final value (MW)</th>
<th>Reconstruction time (s) (After reconnection at t=1s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 9.a</td>
<td>( P_{L1} )</td>
<td>0.7</td>
<td>0.64</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>( P_{G1} )</td>
<td>0.63</td>
<td>-0.12</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>( P_1 )</td>
<td>0.07</td>
<td>0.76</td>
<td>0.07</td>
</tr>
<tr>
<td>Figure 9.b</td>
<td>Voltage droop at PCC1 (%)</td>
<td>Voltage magnitude, KV</td>
<td>Initial value</td>
<td>Intermediate value (in islanded mode)</td>
</tr>
<tr>
<td></td>
<td>2.7%</td>
<td></td>
<td>5</td>
<td>4.875</td>
</tr>
</tbody>
</table>

CONCLUSIONS

This paper proposes a method to load sharing and compensating unbalance load in distributed microgrid. This method compensated the local unbalanced and non-linear loads using state feedback control of switching converter. The DGs considered being ideal dc sources which are connected to the utility using a voltage source converter. In the state feedback controller, states of the system are compared with their reference values to generate the converter switching and these states follow the reference values using linear quadratic regulator (LQR).

Both Grid-connected and island modes responses are considered. System responses in both condition and transfer between the islanded and grid connected mode insures a stable operation of the system and shows high efficiency of the controller. Power sharing and load balancing in a smooth transfer between grid-connected and island mode done with some delay and have some overshoot and under shoot, we can improve power sharing and transient operation of the system with designing PID controller at state feedback controller.

REFERENCES